**PRIMARY VALUES OF INDICES.**

**We know that** $\sqrt[3]{8 } or 8^{\frac{1}{3}}=2$

**Did YOU know that** $\sqrt[3]{-8 } or (-8)^{\frac{1}{3}} \ne -2$

***We say that x2 = 9 has two solutions namely 3 and –3***

***But we also say*** $\sqrt{9}$ ***= 3 but NOT*** $\sqrt{9}$ ***= –3***

***Similarly, the equation x4 = 1 has FOUR solutions, namely 1, i, –1 and –i***

***But we say that*** $\sqrt[4]{1}$ ***= 1 but NOT*** $\sqrt[4]{1} $ ***= 1, i, –1 and –i***

***When we find*** $\sqrt[2]{x}$ ***or*** $\sqrt[3]{x}$ ***or*** $\sqrt[4]{x}$ ***or*** $\sqrt[5]{x}$ ***etc. there is only ONE answer for each root and it is called the PRIMARY ROOT which is the 1st root found when solving xn = b using De Moivres Theorem.***

**Consider the equation *x3 = 8*** which we know has 3 solutions not just the obvious solution ***x = 2***

If we use De Moivre’s Theorem to solve this we proceed as follows:

 ***x3 = 8***

***( r cis(θ) )3 = 8***

 ***r3 cis(3θ) = 8cis( 0 + 360n)***

***r3 = 8 and 3θ = 360n***

***r = 2 and θ = 0 + 120n = 00, 1200, 2400***

***x1 = 2cis( 0 ) = 2***

***x2 = 2cis(120) = – 1 + i√3***

***x3 = 2cis(240) = – 1 – i√3***

***I will refer to x1 as the PRIMARY SOLUTION.***

***The other 2 solutions are generated from this solution by adding multiples of 1200 to the “argument”.***

**So we say that** $\sqrt[3]{8 } or 8^{\frac{1}{3}}=2$

**Now consider *x3 = – 8***

**It “seems” we can just say *x = –2* because (– 2)3 = –8 but – 2 is not the Primary Solution!**

Using De Moivre’s theorem again:

 ***x3 = – 8***

***( r cis(θ) )3 = –8***

 ***r3 cis(3θ) = 8cis( 180 + 360n)***

***r3 = 8 and 3θ = 180 + 360n***

***r = 2 and θ = 60 + 120n = 600, 1800, 3000***

***x1 = 2cis( 60 ) = 1 + i√3***

***x2 = 2cis(180) = – 2***

***x3 = 2cis(240) = 1 – i√3***

**The Primary Solution is** ***x1 =*** ***1 + i√3 ≈ 1 + 1.732i***

**So** $\sqrt[3]{-8 } or (-8)^{\frac{1}{3}}=1+1.732i and NOT-2!$

***The other 2 solutions are generated from this solution by adding multiples of 1200 to the “argument”.***

NB If we type $\sqrt[3]{-8 } or (-8)^{\frac{1}{3}}$ on the graphics calculator we get ***1 + 1.732i*** and not ***–***2

 **Similarly, let us consider *x4 = 1***

 ***( r cis(θ) )4 = 1***

 ***r4 cis(4θ) = 1cis( 0 + 360n)***

***r4 = 1 and 4θ = 360n***

***r = 1 and θ = 00, 900, 1800, 2700***

***x1 = cis( 0 ) = 1***

***x2 = cis(90) = i***

***x3 = cis(180) = – 1***

***x4 = cis(270) = –i***

**The Primary Solution is** ***x1 = 1***

***So that*** $\sqrt[4]{1 } or 1^{\frac{1}{4}}= $***1***

***The other 3 solutions are generated from this solution by adding multiples of 900 to the “argument”.***

**Compare this with *x4 = – 1***

***( r cis(θ) )4 = –1***

 ***r4 cis(4θ) = 1cis(180 + 360n)***

***r4 = 1 and 4θ = 180 + 360n***

***r = 1 and θ = 45 + 90n = 450, 1350, 2250, 3150***

***x1 = cis( 0 ) = cos45 + isin45 = 0.707 + i0.707***

Notice that none of these solutions is a real number!

***x2 = cis(90) = cos135 + isin135 = –0.707 + i0.707***

***x3 = cis(180) = cos225 + isin225 = –0.707 – i0.707***

***x4 = cis(270) = cos315 + isin315 = 0.707 – i0.707***

**The Primary Solution is *x1 =* *0.707 + i0.707***

***The other 3 solutions are generated from this solution by adding multiples of 900 to the “argument”.***

***So that*** $\sqrt[4]{-1 } or (-1)^{\frac{1}{4}}=$ ***0.707 + i0.707 which is verified by the graphics calculator.***

**Consider *x5 = 32***

***( r cis(θ) )5 = 32***

 ***r5 cis(5θ) = 32cis( 0 + 360n)***

***r5 = 32 and 5θ = 360n***

***r = 2 and θ = 72n = 00, 720, 1440, 2160, 2880***

***x1 = 2cis( 0 ) = 2cos0 + 2isin0 = 2***

***x2 = 2cis(72) = 2cos72 + 2isin72 = 0.62 + 1.9i***

***x3 = 2cis(144) = 2cos144 + 2isin144 = – 1.62 + 1.18i***

***x4 = 2cis(216) = 2cos216 + 2isin 216 = – 1.62 – 1.18***

***x5 = 2cis (288) = 2cos288 + 2isin288 = 0.61 – 1.9i***

**The Primary Solution is *x1 = 2***

***The other 4 solutions are generated from this solution by adding multiples of 720 to the “argument”.***

***So that*** $\sqrt[5]{32 } or 32^{\frac{1}{5}}=$ ***2 which is verified by the graphics calculator.***

**Consider *x5 = –32***

***( r cis(θ) )5 = –32***

 ***r5 cis(5θ) = 32cis( 180 + 360n)***

***r5 = 32 and 5θ = 180 + 360n***

***r = 2 and θ = 36 + 72n = 360, 1080, 1800, 2520, 3240***

***x1 = 2cis(36 ) = 2cos36 + 2isin36 = 1.62 + 1.18i***

***x2 = 2cis(108) = 2cos108 + 2isin108 = –0.62 + 1.9i***

***x3 = 2cis(180) = 2cos180+ 2isin180 = –2***

***x4 = 2cis(252) = 2cos252 + 2isin 252 = – 0.62 – 1.9***

***x5 = 2cis (324) = 2cos324+ 2isin324 = 1.62 – 1.18i***

**The Primary Solution is** ***x1 = 1.62 + 1.18i***

***The other 4 solutions are generated from this solution by adding multiples of 720 to the “argument”.***

***So that*** $\sqrt[5]{-32 } or (-32)^{\frac{1}{5}}=$ ***1.62 + 1.18i which is verified by the graphics calculator.***

If we go right back to ***x2 = 9***

***( r cis(θ) )2 = 9***

 ***r2 cis(2θ) = 9cis( 0 + 360n)***

***r2 = 9 and 2θ = 360n***

***r = 3 and θ = 00, 1800***

***x1 = 3cis( 0 ) = 3***

***x2 = 3cis(180) = – 3***

***so √9 = 3 because it is the primary solution,***

***not because it is “a positive number” nor any other reason.***

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***The equation y5 = –32 is not the same as y =***$\sqrt[5]{-32}$

***The equation y5 = –32 has 5 solutions***

***but y =***$\sqrt[ 5]{-32} $***only has 1 solution (the primary solution)***

***Similarly:***

***y2 = 9 has 2 solutions y = +3 or –3***

***but y = 9 ½ only has 1 solution y = +3***