**PRIMARY VALUES OF INDICES.**

**We know that**

**Did YOU know that**

***We say that x2 = 9 has two solutions namely 3 and –3***

***But we also say = 3 but NOT = –3***

***Similarly, the equation x4 = 1 has FOUR solutions, namely 1, i, –1 and –i***

***But we say that = 1 but NOT = 1, i, –1 and –i***

***When we find or or or etc. there is only ONE answer for each root and it is called the PRIMARY ROOT which is the 1st root found when solving xn = b using De Moivres Theorem.***

**Consider the equation *x3 = 8*** which we know has 3 solutions not just the obvious solution ***x = 2***

If we use De Moivre’s Theorem to solve this we proceed as follows:

***x3 = 8***

***( r cis(θ) )3 = 8***

***r3 cis(3θ) = 8cis( 0 + 360n)***

***r3 = 8 and 3θ = 360n***

***r = 2 and θ = 0 + 120n = 00, 1200, 2400***

***x1 = 2cis( 0 ) = 2***

***x2 = 2cis(120) = – 1 + i√3***

***x3 = 2cis(240) = – 1 – i√3***

***I will refer to x1 as the PRIMARY SOLUTION.***

***The other 2 solutions are generated from this solution by adding multiples of 1200 to the “argument”.***

**So we say that**

**Now consider *x3 = – 8***

**It “seems” we can just say *x = –2* because (– 2)3 = –8 but – 2 is not the Primary Solution!**

Using De Moivre’s theorem again:

***x3 = – 8***

***( r cis(θ) )3 = –8***

***r3 cis(3θ) = 8cis( 180 + 360n)***

***r3 = 8 and 3θ = 180 + 360n***

***r = 2 and θ = 60 + 120n = 600, 1800, 3000***

***x1 = 2cis( 60 ) = 1 + i√3***

***x2 = 2cis(180) = – 2***

***x3 = 2cis(240) = 1 – i√3***

**The Primary Solution is** ***x1 =*** ***1 + i√3 ≈ 1 + 1.732i***

**So**

***The other 2 solutions are generated from this solution by adding multiples of 1200 to the “argument”.***

NB If we type  on the graphics calculator we get ***1 + 1.732i*** and not ***–***2

**Similarly, let us consider *x4 = 1***

***( r cis(θ) )4 = 1***

***r4 cis(4θ) = 1cis( 0 + 360n)***

***r4 = 1 and 4θ = 360n***

***r = 1 and θ = 00, 900, 1800, 2700***

***x1 = cis( 0 ) = 1***

***x2 = cis(90) = i***

***x3 = cis(180) = – 1***

***x4 = cis(270) = –i***

**The Primary Solution is** ***x1 = 1***

***So that 1***

***The other 3 solutions are generated from this solution by adding multiples of 900 to the “argument”.***

**Compare this with *x4 = – 1***

***( r cis(θ) )4 = –1***

***r4 cis(4θ) = 1cis(180 + 360n)***

***r4 = 1 and 4θ = 180 + 360n***

***r = 1 and θ = 45 + 90n = 450, 1350, 2250, 3150***

***x1 = cis( 0 ) = cos45 + isin45 = 0.707 + i0.707***

Notice that none of these solutions is a real number!

***x2 = cis(90) = cos135 + isin135 = –0.707 + i0.707***

***x3 = cis(180) = cos225 + isin225 = –0.707 – i0.707***

***x4 = cis(270) = cos315 + isin315 = 0.707 – i0.707***

**The Primary Solution is *x1 =* *0.707 + i0.707***

***The other 3 solutions are generated from this solution by adding multiples of 900 to the “argument”.***

***So that 0.707 + i0.707 which is verified by the graphics calculator.***

**Consider *x5 = 32***

***( r cis(θ) )5 = 32***

***r5 cis(5θ) = 32cis( 0 + 360n)***

***r5 = 32 and 5θ = 360n***

***r = 2 and θ = 72n = 00, 720, 1440, 2160, 2880***

***x1 = 2cis( 0 ) = 2cos0 + 2isin0 = 2***

***x2 = 2cis(72) = 2cos72 + 2isin72 = 0.62 + 1.9i***

***x3 = 2cis(144) = 2cos144 + 2isin144 = – 1.62 + 1.18i***

***x4 = 2cis(216) = 2cos216 + 2isin 216 = – 1.62 – 1.18***

***x5 = 2cis (288) = 2cos288 + 2isin288 = 0.61 – 1.9i***

**The Primary Solution is *x1 = 2***

***The other 4 solutions are generated from this solution by adding multiples of 720 to the “argument”.***

***So that 2 which is verified by the graphics calculator.***

**Consider *x5 = –32***

***( r cis(θ) )5 = –32***

***r5 cis(5θ) = 32cis( 180 + 360n)***

***r5 = 32 and 5θ = 180 + 360n***

***r = 2 and θ = 36 + 72n = 360, 1080, 1800, 2520, 3240***

***x1 = 2cis(36 ) = 2cos36 + 2isin36 = 1.62 + 1.18i***

***x2 = 2cis(108) = 2cos108 + 2isin108 = –0.62 + 1.9i***

***x3 = 2cis(180) = 2cos180+ 2isin180 = –2***

***x4 = 2cis(252) = 2cos252 + 2isin 252 = – 0.62 – 1.9***

***x5 = 2cis (324) = 2cos324+ 2isin324 = 1.62 – 1.18i***

**The Primary Solution is** ***x1 = 1.62 + 1.18i***

***The other 4 solutions are generated from this solution by adding multiples of 720 to the “argument”.***

***So that 1.62 + 1.18i which is verified by the graphics calculator.***

If we go right back to ***x2 = 9***

***( r cis(θ) )2 = 9***

***r2 cis(2θ) = 9cis( 0 + 360n)***

***r2 = 9 and 2θ = 360n***

***r = 3 and θ = 00, 1800***

***x1 = 3cis( 0 ) = 3***

***x2 = 3cis(180) = – 3***

***so √9 = 3 because it is the primary solution,***

***not because it is “a positive number” nor any other reason.***

***\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_***

***The equation y5 = –32 is not the same as y =***

***The equation y5 = –32 has 5 solutions***

***but y =only has 1 solution (the primary solution)***

***Similarly:***

***y2 = 9 has 2 solutions y = +3 or –3***

***but y = 9 ½ only has 1 solution y = +3***