## **BASIC QUADRATIC EQUATIONS**

The logical, thinking method is obviously as follows:

The BASIC IDEA: If  $a \times b = 0$ then a = 0 or b = 0

so if (x-5)(x+8) = 0

*then* x - 5 = 0 *or* x + 8 = 0

so x = 5 or x = -8

But of course, most people soon start to "miss out" the middle step and just write:

$$(x-5)(x+8) = 0$$
  
so  $x = 5$  or  $x = -8$ 

Unfortunately, many students forget the logical reasoning behind this idea and start to make up their own "rule".

Basically, they say "The answers are the numbers in the brackets with the sign changed"!

This of course works for cases like the above.

eg 
$$x^{2} - x - 12 = 0$$
  
 $(x - 4)(x + 3) = 0$   
 $x = 4 \text{ or } x = -3$ 

ie just by using the numbers in the bracket with the signs changed.

Then as soon as they meet questions like the following, everything goes wrong!

If 
$$(3x - 12)(2x + 10) = 0$$
  
then  $3x - 12 = 0$  or  $2x + 10 = 0$   
 $3x = 12$  or  $2x = -10$   
 $x = 4$  or  $x = -5$ 

We then go back to using:

If  $a \times b = 0$  then a = 0 or b = 0

Another problem arises when we solve equations such as:

 $2x^2 + 2x - 24 = 0$  which factorises into 2(x-3)(x+4) = 0

Some students cannot deal with what the "2" means!

They often put x = 3 or -4 or 2!

Again, we have to go back to basics to explain the following:

If  $a \times b \times c = 0$  then a = 0 or b = 0 or c = 0but if  $5 \times b \times c = 0$  we know that  $5 \neq 0$  but we say b could be 0 or c could be 0 Similarly if 8(x-5)(x+7) = 0we know  $8 \neq 0$  so x = 5 or x = -7

Another problem occurs if the equation is of the form  $x^2 - 3x = 0$ 

On factorising this we get: x(x-3) = 0This is exactly the same as  $a \times b = 0$ so of course x = 0 or x = 3

An equation like the following can be immensely instructive:

$$6x(x-4)(x+7)(2x-10) = 0$$
  
is like:  
$$a \times b \times c \times d \times e = 0$$

which is like

(we know that  $6 \neq 0$ ) but x = 0 or x = 4 or x = -7 or x = 5

Hopefully, if students constantly remember the basic idea that if

 $a \times b = 0$  then a = 0 or b = 0

then they will have no problem in fully "understanding" how to solve:

$$6x^{3} - 6x^{2} - 12x = 0$$
 which can be written as  
 $6x(x^{2} - x - 2) = 0$   
 $6x(x - 2)(x + 1) = 0$   
so  $x = 0$  or  $x = 2$  or  $x = -1$