WHY ARE WE STILL MESSING ABOUT WITH SECS?

I recently found a 1946 copy of Cambridge Four Figure Mathematical Tables and noticed there was a table of SECANTS as well as Sines, Cosines and Tangents.

NATURAL SECANTS 13

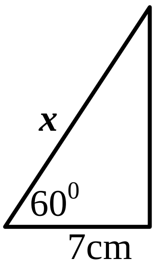
To find cosec $A°$, use cosec $A° = \sec(90-A)°$, e.g. cosec 42° 27′ = sec 47° 33′ = 1·4816

	0′	10′	20′	30′	40′	50′	Differences 1′	2′	3′	4′	5′	6′	7′	8′	9′
45°	1·4142	1·4183	1·4225	1·4267	1·4310	1·4352	4	8	13	17	21	25	30	34	38
46	1·4396	1·4439	1·4483	1·4527	1·4572	1·4617	4	9	13	18	22	27	31	36	40
47	1·4663	1·4709	1·4755	1·4802	1·4849	1·4897	5	9	14	19	23	28	33	38	42
48	1·4945	1·4993	1·5042	1·5092	1·5141	1·5192	5	10	15	20	25	30	35	40	45
49	1·5243	1·5294	1·5345	1·5398	1·5450	1·5504	5	10	16	21	26	31	37	42	47
50°	1·5557	1·5611	1·5666	1·5721	1·5777	1·5833	6	11	17	22	28	33	39	44	50
51	1·5890	1·5948	1·6005	1·6064	1·6123	1·6183	6	12	18	24	29	35	41	47	53
52	1·6243	1·6303	1·6365	1·6427	1·6489	1·6553	6	12	19	25	31	37	44	50	56
53	1·6616	1·6681	1·6746	1·6812	1·6878	1·6945	7	13	20	26	33	40	46	53	59
54	1·7013	1·7081	1·7151	1·7221	1·7291	1·7362	7	14	21	28	35	42	49	56	63
55	1·7434	1·7507	1·7581	1·7655	1·7730	1·7806	7	15	22	30	37	45	52	60	67
56	1·7883	1·7960	1·8039	1·8118	1·8198	1·8279	8	16	24	32	40	48	56	64	72
57	1·8361	1·8443	1·8527	1·8612	1·8697	1·8783	9	17	26	34	43	51	60	68	77
58	1·8871	1·8959	1·9048	1·9139	1·9230	1·9323	9	18	27	36	45	55	64	73	82
59	1·9416	1·9511	1·9606	1·9703	1·9801	1·9900	10	19	29	39	49	58	68	78	88
60°	2·000	2·010	2·020	2·031	2·041	2·052	1	2	3	4	5	6	7	8	9

Sec

I assume that in those days they used five or six trigonometric ratios instead of just the three that we use today. If so, finding the hypotenuse of a triangle could have been done without transposing the equation.

eg



Today's Method:	**1946 Method:**
$\cos 60 = \dfrac{7}{x}$	$\dfrac{x}{7} = \sec 60$
$x = \dfrac{7}{\cos 60}$	$x = 7\sec 60$
$x = \dfrac{7}{0.5}$	$x = 7 \times 2$
$x = 14$ cm	$x = 14$ cm

It then occurred to me, why do we even bother using ***sec, cosec*** and ***cot*** in our Year 13 Calculus course? In fact why does ANYONE still use them?

We do not actually need these archaic quantities at all!

The formula sheet tells us that the derivative of ***tan x*** is $\boldsymbol{\sec^2 x}$.
To be quite realistic, this result means very little to a normal 17 year old.
(Just ask a typical student to work out $\sec^2(\pi/4)$ and you will very probably be confronted by a blank expression.)
What is wrong with putting the derivative of ***tan x*** $= \dfrac{1}{\cos^2 x}$?

This is far more meaningful.
In fact, we should be concentrating on teaching students WHY this is true, NOT just finding the result on a formula sheet!

If $y = \tan x = \dfrac{\sin x}{\cos x}$

$$\frac{dy}{dx} = \frac{\cos x\,(\cos x) - \sin x\,(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

The Differentiation table on the formula sheet should just be as follows:

$y = f(x)$	$\dfrac{dy}{dx} = f\,'(x)$
$\ln(x)$	$\dfrac{1}{x}$
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\dfrac{1}{\cos^2 x}$

There should be no mention of ***sec x*** becoming ***sec x tan x*** etc
If we require the derivative of $\dfrac{1}{\cos x}$ then we should just differentiate it!

eg $y = (\cos x)^{-1}$

$$\frac{dy}{dx} = -(\cos x)^{-2} \times (-\sin x) = \frac{\sin x}{\cos^2 x}$$

(Of course, this equals ***sec x tan x*** but is in a far more meaningful form!)

If anyone is concerned about integrals such as :

$$\int sec\ x\ tan\ x\ \ dx$$

you should remember that **sec, cosec** and **cot** are basically **redundant, archaic quantities**.

The above integral can actually be written as:

$$\int \frac{\sin x}{(cosx)^2}\ \ dx$$

We simply let ***u = cos x***

du = – sinx dx

and the integral becomes: $\int -\frac{du}{u^2}$

$$= \int - u^{-2}\ dx$$

$$= \frac{1}{u} + c$$

$$= \frac{1}{\cos x} + c$$

The above expression was integrated using the simple substitution ***u = cos x***
In fact some people even do this mentally. Either way, this is an integral **worthy of being in a calculus course.**
However, in its original form, $\int sec\ x\ tan\ x\ \ dx$ what is the benefit in just looking at the formula sheet and writing: $\int sec\ x\ tan\ x\ \ dx$ = ***sec x*** + c