**WHY DOES NEGATIVE TIMES NEGATIVE BECOME POSITIVE?**

This is a classic example of “knowing” is not “understanding”.

Most people “KNOW” that **– 3 × –4 = +12** but very few can explain WHY.

I even asked some students this very question and the popular answer was simply that “IT IS THE RULE!”

I find it very sad that most “explanations” of why negative **×** negative = positive

just do not remain in people’s brains.

There are many “novel” ways such as: walking forwards (for positive) and backwards (for negative) then turning through 1800 and walking backwards which results in a positive again. However these ideas rarely last long in people’s minds.

One method I like to use for 12 year olds requires the following ideas:

(a) **3 × 4 means 4 + 4 + 4**

 **4 × 3 means 3 + 3 + 3 + 3**

The fact that they both equal **12** is not the point.

 **3 × 4 and 4 × 3 actually mean different things but give the same result.**

(b) **3 × –4 therefore means –4 +  –4 +  –4 = –12**

 **So we could say that positive × negative = negative but I want people**

 **to remember WHY and not just remember the RULE!**

(c) Students need to grasp the idea of **“opposites”** like **+3** and **–3**

 and that **+3** **+** **–3 = 0**

(d) The next bit is a little unsatisfactory but still effective.

 If we “put” a negative in front of a number, it becomes its opposite and it

 helps to say the word “opposite” instead of “negative”.

 So the opposite of **+3** is **–3**

 Also the opposite of **–3 is written as – (–3) which of course is +3**

(e) If we consider **– 3 × –4** we could think of the first negative as detachable

 and put **– ( 3 × –4 )** ie the **opposite of (3 × –4 )**

 = the **opposite of (– 12)**

 = **+ 12**

 **Hence**  **– 3 × –4 = +12**

We could even think of “detaching” both negatives and think of

 **– 3 × –4 as – (– (3 ×4) ) which is the opposite of the opposite of 3 × 4**

 which of course equals **+12.**

 **This is also an effective way of realising that any EVEN number of**

 **negatives produces a POSITIVE answer and any ODD number of**

 **negatives produces a NEGATIVE answer.**

**For slightly older students, the following method is by far the best.**

We know from the opposites idea that **+3** **+**  **–3**  **=** 0

so multiplying both sides by **–4 produces: –4 (+3** **+**  **–3 ) = –4 × 0**

If we expand the bracket we get: **–4 ×+3** **+ –4 ×** **–3 =**  0

We know that **–4 ×+3** = **–12 so we now have the equation:**

 **–12 + –4 ×** **–3 =**  0

but we know from the opposites idea that **–12 +  + 12 =**  0

By comparing these two equations **–4 ×** **–3 MUST BE + 12**

 **NEGATIVE × NEGATIVE = POSITIVE**

From the point of view that UNDERSTANDING is the most important goal, a numerical demonstration is often much better than a general algebraic version.

 **The following general method is mathematically nicer,**

 **but would not be as effective as the above.**

We know from the opposites idea that***+b******+*** ***–b*** ***=*** *0*

so multiplying both sides by **–a produces: *–a (+b******+*** ***–b ) = –a × 0***

If we expand the bracket we get: ***–a ×+b******+ –a ×******–b =***  *0*

We know that ***–a × +b*** *=* ***–ab* so we now have the equation:**

 ***–ab + –a ×******–b =***  *0*

but we know from the opposites idea that ***–ab +  +ab =***  *0*

By comparing these two equations ***–a ×******–b* MUST BE *+ab***

 **NEGATIVE × NEGATIVE = POSITIVE**