WHY DOES NEGATIVE TIMES NEGATIVE BECOME POSITIVE?

This is a classic example of "knowing" is not "understanding".
Most people "KNOW" that **$^{-}3 \times {}^{-}4 = {}^{+}12$** but very few can explain WHY.
I even asked some students this very question and the popular answer was simply that "IT IS THE RULE!"
I find it very sad that most "explanations" of why negative × negative = positive just do not remain in people's brains.
There are many "novel" ways such as: walking forwards (for positive) and backwards (for negative) then turning through 180^0 and walking backwards which results in a positive again. However these ideas rarely last long in people's minds.

One method I like to use for 12 year olds requires the following ideas:

(a) **3 × 4 means 4 + 4 + 4**
4 × 3 means 3 + 3 + 3 + 3
The fact that they both equal **12** is not the point.
3 × 4 and 4 × 3 actually <u>mean</u> different things but give the same result.

(b) **$3 \times {}^{-}4$ therefore means $^{-}4 + {}^{-}4 + {}^{-}4 = {}^{-}12$**
So we could say that positive × negative = negative but I want people to remember WHY and not just remember the RULE!

(c) Students need to grasp the idea of **"opposites"** like **$^{+}3$** and **$^{-}3$**
and that **$^{+}3 + {}^{-}3 = 0$**

(d) The next bit is a little unsatisfactory but still effective.
If we "put" a negative in front of a number, it becomes its opposite and it helps to say the word "opposite" instead of "negative".
So the opposite of **$^{+}3$** is **$^{-}3$**
Also the opposite of **$^{-}3$ is written as $^{-}(^{-}3)$ which of course is $^{+}3$**

(e) If we consider **$^{-}3 \times {}^{-}4$** we could think of the first negative as detachable and put **$^{-}(3 \times {}^{-}4)$** ie the **opposite of $(3 \times {}^{-}4)$**
= the **opposite of $(^{-}12)$**
= **$^{+}12$**

Hence $^{-}3 \times {}^{-}4 = {}^{+}12$
We could even think of "detaching" both negatives and think of
$^{-}3 \times {}^{-}4$ as $^{-}(^{-}(3 \times 4))$ which is the opposite of the opposite of 3 × 4
which of course equals **$^{+}12$.**
This is also an effective way of realising that any EVEN number of negatives produces a POSITIVE answer and any ODD number of negatives produces a NEGATIVE answer.

For slightly older students, the following method is by far the best.

We know from the opposites idea that $\quad {}^{+}\mathbf{3} \;+\; {}^{-}\mathbf{3} \;=\; 0$

so multiplying both sides by ${}^{-}$**4 produces:** $\;{}^{-}\mathbf{4}\,({}^{+}\mathbf{3} \;+\; {}^{-}\mathbf{3}) \;=\; {}^{-}\mathbf{4} \times \mathbf{0}$

If we expand the bracket we get: $\quad {}^{-}\mathbf{4} \times {}^{+}\mathbf{3} \;+\; {}^{-}\mathbf{4} \times {}^{-}\mathbf{3} \;=\; 0$

We know that ${}^{-}\mathbf{4} \times {}^{+}\mathbf{3} = {}^{-}\mathbf{12}$ **so we now have the equation:**

$${}^{-}\mathbf{12} \;+\; {}^{-}\mathbf{4} \times {}^{-}\mathbf{3} \;=\; 0$$

but we know from the opposites idea that $\quad {}^{-}\mathbf{12} \;+\; {}^{+}\mathbf{12} \;=\; 0$

By comparing these two equations ${}^{-}\mathbf{4} \times {}^{-}\mathbf{3}$ **MUST BE** ${}^{+}\mathbf{12}$

NEGATIVE × NEGATIVE = POSITIVE

From the point of view that UNDERSTANDING is the most important goal, a numerical demonstration is often much better than a general algebraic version.

The following general method is mathematically nicer, but would not be as effective as the above.

We know from the opposites idea that $\quad {}^{+}b \;+\; {}^{-}b \;=\; 0$

so multiplying both sides by ${}^{-}$**a produces:** $\;{}^{-}a\,({}^{+}b \;+\; {}^{-}b) \;=\; {}^{-}a \times 0$

If we expand the bracket we get: $\quad {}^{-}a \times {}^{+}b \;+\; {}^{-}a \times {}^{-}b \;=\; 0$

We know that ${}^{-}a \times {}^{+}b = {}^{-}ab$ **so we now have the equation:**

$${}^{-}ab \;+\; {}^{-}a \times {}^{-}b \;=\; 0$$

but we know from the opposites idea that $\quad {}^{-}ab \;+\; {}^{+}ab \;=\; 0$

By comparing these two equations ${}^{-}a \times {}^{-}b$ **MUST BE** ${}^{+}ab$

NEGATIVE × NEGATIVE = POSITIVE