TRICKY POINTS DEALING WITH SURDS.

Many students perpetually make the mistake of writing things like:

$$\sqrt{(x^2\ +\ y^2)}\ = \boldsymbol{x} + \boldsymbol{y}$$

They simply need to be shown what we can and cannot do by considering problems which can be worked out two ways so that we can verify certain procedures.

Consider $\sqrt{16 + \ 9}$

If we do the brackets first, we get √**25** which of course equals **5**

Then we **TEST** whether we can do the following:

$$\begin{aligned} \sqrt{16 + \ 9} &= \sqrt{(16)} \ + \ \sqrt{(9)} \\ &= \ 4 \ \ + \ \ 3 \\ &= \ 7 \end{aligned}$$

So it appears that $\sqrt{(a + b)} \neq \sqrt{a} + \sqrt{b}$

It still **"looks"** very tempting to put: $\sqrt{(x^2\ +\ y^2)}\ = \boldsymbol{x} + \boldsymbol{y}$
but the only way the answer can be $\boldsymbol{x} + \boldsymbol{y}$ is as follows!

$$\sqrt{(x^2\ + 2xy +\ y^2)}\ = \sqrt{(x+y)^2} = \boldsymbol{x} + \boldsymbol{y}$$

Students should satisfy themselves about what we can or cannot do with other numerical cases by checking both ways as follows:

$\sqrt{100 - 64}$ = √ **(36) = 6** which is correct!

but $\sqrt{100 - 64}$ = √**(100)** – √**(36) = 10 – 6 = 4** which is clearly wrong!

<u>Have you ever seen this point addressed in any text?</u>

DOES $\sqrt{a \times b}$ *always equal* $\sqrt{a} \times \sqrt{b}$?

This is all about the **ORDER** in which the calculations are done.

$\sqrt{a \times b}$ means multiply ***a*** and ***b*** **THEN** find the square root.
$\sqrt{a} \times \sqrt{b}$ means find each square root **THEN** multiply them.

Sometimes they are not equal!

<u>These examples will show when.</u>

Consider $\sqrt{4} \times \sqrt{9} = 2 \times 3 = 6$
and $\sqrt{(4 \times 9)} = \sqrt{36} = 6$
So when both numbers are positive $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$

Now consider $\sqrt{4} \times \sqrt{(-9)} = 2 \times 3i = 6i$
and $\sqrt{4 \times (-9)} = \sqrt{-36} = 6i$
So when one number is positive and the other is negative $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$

Now consider $\sqrt{(-4)} \times \sqrt{(-9)} = 2i \times 3i = -6$
BUT $\sqrt{(-4) \times (-9)} = \sqrt{36} = +6$
So when both numbers are **negative** $\sqrt{a \times b} \neq \sqrt{a} \times \sqrt{b}$

This is the source of many false proofs for example:

$$1 = \sqrt{+1} = \sqrt{(-1) \times (-1)} = \sqrt{(-1)} \times \sqrt{(-1)} = i \times i = -1$$

This seems to prove 1 = –1 but the false step is in RED type!

We should also consider if $\sqrt{\dfrac{\boldsymbol{a}}{\boldsymbol{b}}} = \dfrac{\sqrt{\boldsymbol{a}}}{\sqrt{\boldsymbol{b}}}$

$\sqrt{\dfrac{a}{b}}$ means "find ***a*** divided by ***b*** first **then** find the square root"

while $\dfrac{\sqrt{a}}{\sqrt{b}}$ means "find the square roots of ***a*** and ***b*** first **then** divide them".

Example 1.

$\sqrt{\dfrac{64}{16}} = \sqrt{4} = 2$	$\dfrac{\sqrt{64}}{\sqrt{16}} = \dfrac{8}{4} = 2$
So $\sqrt{\dfrac{64}{16}} = \dfrac{\sqrt{64}}{\sqrt{16}}$	

Example 2.

$\sqrt{\dfrac{-64}{16}} = \sqrt{-4} = 2i$	$\dfrac{\sqrt{-64}}{\sqrt{16}} = \dfrac{8i}{4} = 2i$
So $\sqrt{\dfrac{-64}{16}} = \dfrac{\sqrt{-64}}{\sqrt{16}}$	

Example 3.

$\sqrt{\dfrac{64}{-16}} = \sqrt{-4} = \boldsymbol{2i}$	$\dfrac{\sqrt{64}}{\sqrt{-16}} = \dfrac{8}{4i} = \dfrac{8}{4i} \times \dfrac{i}{i} = \boldsymbol{-2i}$
So $\sqrt{\dfrac{64}{-16}} \neq \dfrac{\sqrt{64}}{\sqrt{-16}}$	

Example 4.

$\sqrt{\dfrac{-64}{-16}} = \sqrt{4} = \boldsymbol{2}$	$\dfrac{\sqrt{-64}}{\sqrt{-16}} = \dfrac{8i}{4i} = \boldsymbol{2}$
So $\sqrt{\dfrac{-64}{-16}} = \dfrac{\sqrt{-64}}{\sqrt{-16}}$	