**TEACHING INTEGERS.**

 (After 47 years of teaching MANY different ways to teach **Integers**,

 I consider the following method is BY FAR the best.)

**INTRO:** Possible discussion starters:

1. Suppose the temperature is 60 C.

 If the temp drops 70C then we say it is “10 below zero” or “minus 10C” or “ **–10C**

2. If you only have $20 in your account and you write a cheque for $25 then your

 account is referred to as **“$5 in the red”**, meaning **you owe the bank $5**

 or you have **–5 dollars. (*It is* *better to say “negative 5” rather than “minus 5”* )**

**OBJECTIVES.**

1. to know the Integer Number Line.

2. to understand > and < signs.

3. to understand that +3 means **move 3 units to the right** and

 –3means **move 3 units to the left**.

4. to know how to add all combinations of ±a **+** ±b by drawing diagrams and

 mentally.

5. to know how to add all combinations of ±a **–** ±b by drawing diagrams and

 mentally.

6. to know how to multiply and divide integers.

**TEACHING NOTES.**

There are several ways to teach concepts of integers but I strongly recommend the following:

**1**. A good definition of  ***a < b*** is that ***a* is on the left of *b* o**n a number line.

 . . . . . . . . .

 -4 -3 -2 -1 0 +1 +2 +3 +4

 -3 < -2 +2 < +3

 because -3 because +2

 is on the **left** is on the **left**

 of -2 of +3

**2.** Teach students that **+3 means move 3 units to the right** +3

 and that **–3means move 3 units to the left**. –3

**3**. +3 and –3 are called **OPPOSITES**

 –3 is the opposite of +3 and +3 is the opposite of –3

 And **opposites add to zero** ie **–3 ++3 = 0**

**4**. Students find it very tedious drawing number lines to show additions so I suggest

 you give them the sheet of **prepared number lines** to stick in their books, called

 “ADDING INTEGERS” to teach the idea covering all cases of ***±a + ±b.***

**ADDING INTEGERS.**

This diagram shows us WHY

 **+5 +  – 3 = +2** using a number line. **ALWAYS START FROM ZERO**

 -4 -3 -2 -1 0 +1 +2 +3 +4 +5

On the following number lines, show **HOW** the results can be obtained.

 NOTE:

a. **+2 +  + 3 =** +5 **+2 +  + 3**

 is best thought of as

 +2 **followed by** +3

 -4 -3 -2 -1 0 +1 +2 +3 +4 +5

b. **+2 +  – 5 =** **– 3**

 BASICALLY WE ARE

 TREATING ADDITION OF

 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 INTEGERS JUST LIKE

 ADDITION OF VECTORS

 WHERE THE DIRECTION

c. **+4 +  – 1 =** **+ 3** IS **CRUCIAL**.

 They probably do linear

 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 vector problems similar to

 this in Science.

d. **–2 +  + 5 =** **+ 3**

 -4 -3 -2 -1 0 +1 +2 +3 +4 +5

e. **–4 +  + 3 =** **–1**

 -4 -3 -2 -1 0 +1 +2 +3 +4 +5

f. **–2 +  –1 =** **– 3**

 -4 -3 -2 -1 0 +1 +2 +3 +4 +5

**5**. **SUBTRACTION.** We should try to get the idea that +6 **–** +4 is really the same

 as +6 **+** –4 ie **subtracting is adding the opposite**.

**It is quite instructive to think of “ + ” to mean “ *followed by* ”**

**and to think of “ – ” to mean “*followed by the opposite of* ”.**

In the same way that +6 **+** –4 is better thought of as +6 followed by –4

we can think of +6 **–** +4 as +6 ***followed by the opposite******of*** +4

**This idea covers the idea of the double negative beautifully:**

Eg. +4  **–** –3  becomes +4 “***followed by the opposite” of*** –3

 which is +4 and **+3** = +7

Perhaps instead of “***followed by***” you could just use “***and*** ”, and instead of “***followed by the opposite of*** ” you could just use “***and the opposite of*** ”.

egs

a. +5 – +3 think of: +5 ***and opposite of*** +3 = +2

b. +5 – +7 think of: +5 ***and opposite of*** +7 = –2

c. –2 – +3 think of: –2 ***and opposite of*** +3 = –5

d. –2 – –6 think of: –2 ***and opposite of*** –6 = +4

e. –8 – –3 think of –8 ***and opposite of*** –3 = –5

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**6**. Students will keep trying to **MAKE UP THEIR OWN RULES** in cases like these.

 **Their “rules” often do NOT work properly!**

 Watch out for people who try to use things like:

 ***Positive + Negative = Negative*** or something similar !!!

Show clearly there is no such rule because:

 +6+ –2 = **+4** **ie Pos + Neg = Pos this time**

 but +6+ –10 = **–4 ie Pos + Neg = Neg this time.**

**7**. However we DO need to make rules for multiplication and division.

 I like to do it in a **very logical way** as follows:

(a) **+3 × +4** or just **3 × 4** MEANS **3 lots of 4 = 4 + 4 + 4 = 12**

 from which we deduce the obvious that POSITIVE × POSITIVE = POSITIVE

 also **+4 × +3** or just **4 × 3** MEANS 4 lots of 3 = **3+3+3+3 = 12**

 so **4 × 3 and 3 × 4 mean different things but both come to 12.**

(b) Consider **+3× –5** which means **–5 + –5 + –5 = –15**

 from which we deduce the obvious that POSITIVE × NEGATIVE = NEGATIVE

(c) Now consider **–5 ×+3**. We can’t think of **–5 lots** of +3 but we can turn it around

 as in part (a) and call it **+3× –5 = –15**

 from which we deduce the obvious that NEGATIVE × POSITIVE = NEGATIVE

 **BUT before we continue, it is useful to think of –5 ×+3**

 **as : “the opposite ”of 5 × +3 = “the opposite ”of 15 = –15**

(d) **Lastly, consider –4 × –5 This seems a tricky point**

 **Think of “opposite ”of 4 × –5 but well worth doing.**

 **= “opposite ”of –20**

 **= +20**

 **from which we deduce that NEGATIVE × NEGATIVE = POSITIVE**

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A real PROOF that ***–a× – b = +a.b*** is really too hard for young students but here it is:

Obviously ***b + – b = 0***

 So ***– a( b + – b) = 0***

Expanding ***– a×b + – a × – b = 0***

 ***But – a×b + a×b = 0***

This shows us that the **opposite** of ***– a×b*** is ***– a × – b***

 but we know that the **opposite** of ***– a×b*** is ***a×b***

so ***– a ×– b*** must equal ***a×b***

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Since we know that **6 ÷ 2** is really the multiplication **6 × ½** we can say that the

same 4 rules apply to division too.

 **Pos × Pos = Pos**

 **Pos × Neg = Neg**

 **Neg × Pos = Neg**

 **Neg × Neg = Pos**

**ie Pos ÷ Pos = Pos as well as :**

 **Pos ÷ Neg = Neg**

 **Neg ÷ Pos = Neg**

 **Neg ÷ Neg = Pos**

We constantly need to reinforce that they **should NOT apply these rules to addition**.

A **common mistake** is to shorten the rule to “**2 negatives make a positive**”!

You need to reinforce **–4 × –5** = **+20** BUT **–4 + –5** = **–9**

8. Substitution problems can be quite instructive in themselves.

 Suppose ***a = 6 , b = –2 , c = –5***

Apart from finding the usual, ***a + b, b + c , a – b , b – c*** etc.

Students should see things like ***–a*** which should be read as “the opposite of” ***a*** = –6

 and ***–c*** which should be read as “the opposite of” ***c*** = +5

**9**. The difference between **– 42** which is –16 and **( –4)2** which is +16 , is **very**

 **important** particularly when working out ***y*** values such as :

 ***y = 6x – x2* when *x = –4***

 **= 6 × –4 – (­­–4)2**

 **= –24 – (+16)**

 **= –40**

The idea that – 42 has “implied brackets” is difficult to grasp. ie – (42)

**(NB If students just type – 42 on a calculator they will get –16)**

**10**. In multiplications, the idea that “***an EVEN number of negatives means the answer is positive and an ODD number of negatives means a negative answer***”, takes a while for some students to grasp but it makes a good class discussion time discovering this idea.

ie ­–4 ­×–3 × –5 × 2 × 1 = **–120**  but –4 ­× –3 × –5 × 2 × –1 = **+120**

and (–1)25 = **–1** but (–1)26 = **+1**