DEALING WITH SURDS.

Dealing with surds is NOT a matter of using "rules" but of using "logic".

1. Consider: $\sqrt{9} \times \sqrt{9}$

Obviously this is 3×3

$= 9$

But notice that we can do this **in a different order**: $\sqrt{9} \times \sqrt{9}$

$= \sqrt{(9 \times 9)}$

$= \sqrt{(81)}$

$= 9$ as above!

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2. This means that we can write:

$\sqrt{3} \times \sqrt{5} = \sqrt{(3\times5)} = \sqrt{15}$

Or in general : $\sqrt{a}\times\sqrt{b} = \sqrt{(ab)}$

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3. BUT consider $\sqrt{(9 + 16)}$

$= \sqrt{(25)}$

$= 5$

But if we try it **in a different order like before it is not correct:**

Consider $\sqrt{(9 + 16)}$

$= \sqrt{(9)} + \sqrt{(16)}$

$= 3 + 4$

$= 7$

because we know the real answer is 5.

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4. This means that $\sqrt{(a + b)} \neq \sqrt{a} + \sqrt{b}$

So even though it **looks tempting** to do the following:

$\sqrt{(x^2 + 16)} = x + 4$ we cannot do it!

But notice that $\sqrt{(x^2 + 8x + 16)}$

$= \sqrt{(x + 4)^2}$

$= x + 4$

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5. Consider $\sqrt{(6^2 + 8^2)}$

It does look very tempting to put:

$\sqrt{(6^2 + 8^2)} = 6 + 8 = 14$ but we can see that the correct answer is:

$\sqrt{(6^2 + 8^2)} = \sqrt{(36 + 64)}$

$= \sqrt{100}$

$= 10.$

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6. Clearly we can deal with fractions as follows: $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$

so that $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

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7. **Some** surds may be simplified as follows as long as a factor is a perfect square: $\sqrt{48} = \sqrt{16 \times 3}$

$= \sqrt{16} \times \sqrt{3}$

$= 4\times\sqrt{3}$

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8. It is not possible to add $\sqrt{20} + \sqrt{30}$ but $\sqrt{50} + \sqrt{72}$ can be simplified:

$= \sqrt{(25\times2)} + \sqrt{(36\times2)}$

$= 5\sqrt{2} + 6\sqrt{2}$

$= 11\sqrt{2}$

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9. Similarly $\sqrt{63} + \sqrt{28}$

$= \sqrt{9 \times 7} + \sqrt{4 \times 7}$

$= 3\sqrt{7} + 2\sqrt{7}$

$= 5\sqrt{7}$

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10. Surds are easier to deal with if they have **rational** denominators.

The two types are:

(a) $\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

(b) $\frac{5+\sqrt{2}}{4-\sqrt{2}} = \frac{5+\sqrt{2}}{4-\sqrt{2}} \times \frac{4+\sqrt{2}}{4+\sqrt{2}}$

$= \frac{20+4\sqrt{2}+5\sqrt{2}+2}{16+4\sqrt{2}-4\sqrt{2}-2}$

$= \frac{22+9\sqrt{2}}{14}$ or $\frac{22}{14} + \frac{9\sqrt{2}}{14}$

(we usually write irrationals in the form $a + b\sqrt{c}$ rather than $\frac{a+b\sqrt{c}}{d}$)
