TEACHING INTEGERS.

(After 47 years of teaching MANY different ways to teach **Integers**,
I consider the following method is BY FAR the best.)

INTRO: Possible discussion starters:

1. Suppose the temperature is 6^0 C.
   If the temp drops 7^0C then we say it is "1^0 below zero" or "minus 1^0C" or " **$^{-}1^0$C**
2. If you only have \$20 in your account and you write a cheque for \$25 then your account is referred to as **"\$5 in the red"**, meaning **you owe the bank \$5** or you have **$^{-}$5 dollars. (*It is better to say "negative 5" rather than "minus 5"*)**

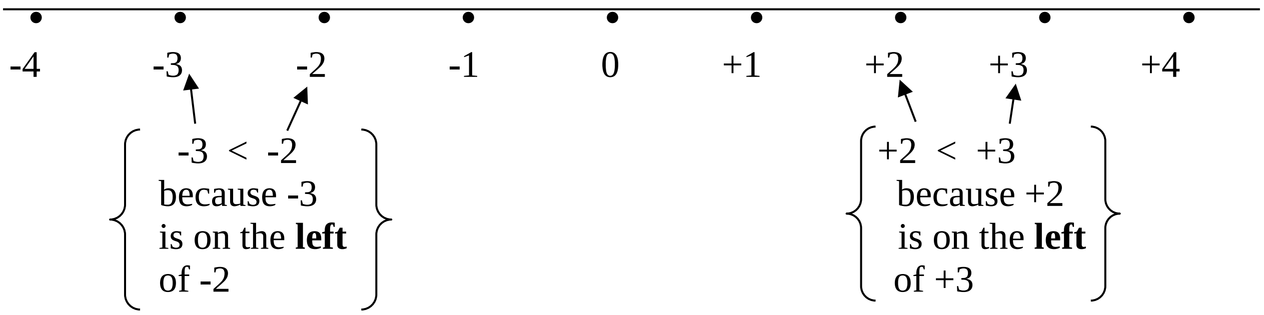
OBJECTIVES.

1. to know the Integer Number Line.
2. to understand > and < signs.
3. to understand that $^{+}3$ means **move 3 units to the right** and
   $^{-}3$ means **move 3 units to the left.**
4. to know how to add all combinations of $^{\pm}a$ **+** $^{\pm}b$ by drawing diagrams and mentally.
5. to know how to add all combinations of $^{\pm}a$ **–** $^{\pm}b$ by drawing diagrams and mentally.
6. to know how to multiply and divide integers.

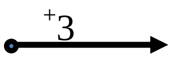
TEACHING NOTES.

There are several ways to teach concepts of integers but I strongly recommend the following:

1. A good definition of ***a* < *b*** is that $\boxed{\textbf{\textit{a} is on the left of \textit{b}}}$ **on** a number line.



2. Teach students that **$^{+}$3 means move 3 units to the right**



and that **$^{-}$3 means move 3 units to the left**.

3. $^{+}3$ and $^{-}3$ are called **OPPOSITES**

$^{-}3$ is the opposite of $^{+}3$ and $^{+}3$ is the opposite of $^{-}3$

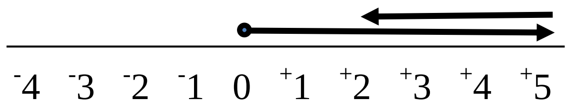
And **opposites add to zero** ie **$^{-}3 + ^{+}3 = 0$**

4. Students find it very tedious drawing number lines to show additions so I suggest you give them the sheet of **prepared number lines** to stick in their books, called "ADDING INTEGERS" to teach the idea covering all cases of $^{\pm}\boldsymbol{a} + {}^{\pm}\boldsymbol{b}$.

<u>ADDING INTEGERS.</u>

This diagram shows us WHY

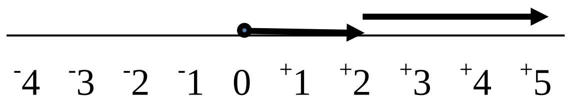
$^{+}\mathbf{5} + {}^{-}\mathbf{3} = {}^{+}\mathbf{2}$ using a number line.



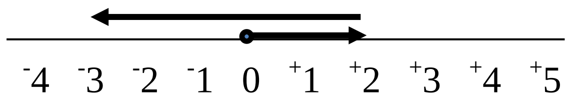
<u>ALWAYS START FROM ZERO</u>

On the following number lines, show **HOW** the results can be obtained.

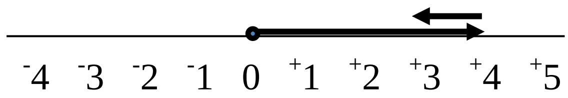
a. $^{+}\mathbf{2} + {}^{+}\mathbf{3} =$ $\boxed{+5}$



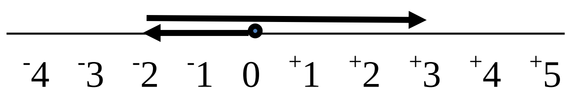
b. $^{+}\mathbf{2} + {}^{-}\mathbf{5} =$ $\boxed{{}^{-}\mathbf{3}}$



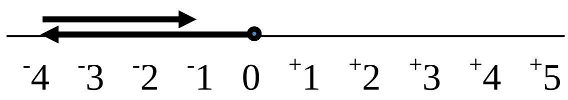
c. $^{+}\mathbf{4} + {}^{-}\mathbf{1} =$ $\boxed{{}^{+}\mathbf{3}}$



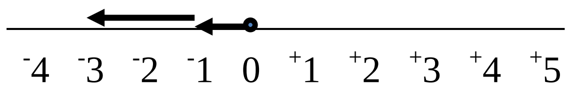
d. $^{-}\mathbf{2} + {}^{+}\mathbf{5} =$ $\boxed{{}^{+}\mathbf{3}}$



e. $^{-}\mathbf{4} + {}^{+}\mathbf{3} =$ $\boxed{{}^{-}\mathbf{1}}$



f. $^{-}\mathbf{2} + {}^{-}\mathbf{1} =$ $\boxed{{}^{-}\mathbf{3}}$



NOTE:

$^{+}\mathbf{2} \quad \mathbf{+} \quad {}^{+}\mathbf{3}$

is best thought of as

+2 **followed by** +3

BASICALLY WE ARE TREATING ADDITION OF INTEGERS JUST LIKE ADDITION OF VECTORS WHERE THE DIRECTION IS **CRUCIAL**.

They probably do linear vector problems similar to this in Science.

5. **SUBTRACTION.** We should try to get the idea that $^{+}6 - {}^{+}4$ is really the same as $^{+}6 + {}^{-}4$ ie **subtracting is adding the opposite**.

It is quite instructive to think of " + " to mean " *followed by* " and to think of " – " to mean "*followed by the opposite of* ".

In the same way that $^{+}6 + {}^{-}4$ is better thought of as $^{+}6$ followed by $^{-}4$
we can think of $^{+}6 - {}^{+}4$ as $^{+}6$ ***followed by the opposite of*** $^{+}4$

This idea covers the idea of the double negative <u>beautifully</u>:

Eg. $^{+}4 - {}^{-}3$ becomes $^{+}4$ "***followed by the opposite" of*** $^{-}3$
which is $^{+}4$ and $^{+}\mathbf{3}$ = $^{+}7$

Perhaps instead of "***followed by***" you could just use "***and*** ", and instead of "***followed by the opposite of*** " you could just use "***and the opposite of*** ".

egs

a. $^{+}5 - {}^{+}3$ think of: $^{+}5$ ***and opposite of*** $^{+}3$ = $^{+}2$

b. $^{+}5 - {}^{+}7$ think of: $^{+}5$ ***and opposite of*** $^{+}7$ = $^{-}2$

c. $^{-}2 - {}^{+}3$ think of: $^{-}2$ ***and opposite of*** $^{+}3$ = $^{-}5$

d. $^{-}2 - {}^{-}6$ think of: $^{-}2$ ***and opposite of*** $^{-}6$ = $^{+}4$

e. $^{-}8 - {}^{-}3$ think of $^{-}8$ ***and opposite of*** $^{-}3$ = $^{-}5$

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6. Students will keep trying to **<u>MAKE UP THEIR OWN RULES</u>** in cases like these.

Their "rules" often do NOT work properly!
Watch out for people who try to use things like:
Positive + Negative = Negative or something similar !!!

Show clearly there is no such rule because:

$^{+}6 + {}^{-}2 = {}^{+}\mathbf{4}$ **ie Pos + Neg = Pos this time**

but $^{+}6 + {}^{-}10 = {}^{-}\mathbf{4}$ **ie Pos + Neg = Neg this time.**

7. However we DO need to make rules for multiplication and division.
I like to do it in a **very logical way** as follows:

(a) $^{+}\mathbf{3} \times {}^{+}\mathbf{4}$ or just $\mathbf{3 \times 4}$ MEANS **3 lots of 4 = 4 + 4 + 4 = 12**
from which we deduce the obvious that POSITIVE × POSITIVE = POSITIVE

also $^{+}\mathbf{4} \times {}^{+}\mathbf{3}$ or just $\mathbf{4 \times 3}$ MEANS 4 lots of 3 = **3+3+3+3 = 12**

so **4 × 3 and 3 × 4 <u>mean different things</u> but both come to 12.**

(b) Consider $^{+}\mathbf{3} \times {}^{-}\mathbf{5}$ which means $^{-}\mathbf{5} + {}^{-}\mathbf{5} + {}^{-}\mathbf{5} = {}^{-}\mathbf{15}$

from which we deduce the obvious that POSITIVE × NEGATIVE = NEGATIVE

(c) Now consider $^{-}\mathbf{5} \times {}^{+}\mathbf{3}$. We can't think of $^{-}$**5 lots** of $^{+}3$ but we can turn it around as in part (a) and call it $^{+}\mathbf{3} \times {}^{-}\mathbf{5} = {}^{-}\mathbf{15}$

from which we deduce the obvious that NEGATIVE × POSITIVE = NEGATIVE

BUT before we continue, it is useful to think of $^{-}\mathbf{5} \times {}^{+}\mathbf{3}$
as : "the opposite "of $\mathbf{5 \times {}^{+}3}$ **= "the opposite "of 15** $= {}^{-}\mathbf{15}$

(d) **Lastly, consider** $^{-}\mathbf{4} \times {}^{-}\mathbf{5}$
Think of "opposite "of $\mathbf{4 \times {}^{-}5}$
= "opposite "of $^{-}\mathbf{20}$
$= {}^{+}\mathbf{20}$

This seems a tricky point but well worth doing.

from which we deduce that NEGATIVE × NEGATIVE = POSITIVE

A real PROOF that $^{-}\boldsymbol{a} \times {}^{-}\boldsymbol{b} = {}^{+}\boldsymbol{a.b}$ is really too hard for young students but here it is:

Obviously $\quad b + {}^{-}b = 0$
So $\quad {}^{-}a(b + {}^{-}b) = 0$
Expanding $\quad {}^{-}a \times b + {}^{-}a \times {}^{-}b = 0$
But $\quad {}^{-}a \times b + a \times b = 0$

This shows us that the **opposite** of $^{-}\boldsymbol{a \times b}$ is $^{-}\boldsymbol{a} \times {}^{-}\boldsymbol{b}$
but we know that the **opposite** of $^{-}\boldsymbol{a \times b}$ is $\boldsymbol{a \times b}$
so $^{-}\boldsymbol{a} \times {}^{-}\boldsymbol{b}$ must equal $\boldsymbol{a \times b}$

Since we know that **6 ÷ 2** is really the multiplication **6 × ½** we can say that the same 4 rules apply to division too.

ie Pos ÷ Pos = Pos
Pos ÷ Neg = Neg
Neg ÷ Pos = Neg
Neg ÷ Neg = Pos

as well as :

Pos × Pos = Pos
Pos × Neg = Neg
Neg × Pos = Neg
Neg × Neg = Pos

We constantly need to reinforce that they **<u>should NOT apply these rules to addition</u>**.

A **common mistake** is to shorten the rule to "**2 negatives make a positive**"!

You need to reinforce $^{-}\mathbf{4} \times {}^{-}\mathbf{5} = {}^{+}\mathbf{20}$ BUT $^{-}\mathbf{4} + {}^{-}\mathbf{5} = {}^{-}\mathbf{9}$

8. Substitution problems can be quite instructive in themselves.

Suppose $a = 6$, $b = {}^{-}2$, $c = {}^{-}5$

Apart from finding the usual, $a + b$, $b + c$, $a - b$, $b - c$ etc.

Students should see things like ^{-}a which should be read as "the opposite of" $a = {}^{-}6$

and ^{-}c which should be read as "the opposite of" $c = {}^{+}5$

9. The difference between $-\mathbf{4}^2$ which is $^{-}16$ and $(^{-}\mathbf{4})^2$ which is $^{+}16$, is **very important** particularly when working out y values such as :

$y = 6x - x^2$ **when** $x = {}^{-}4$

$= \mathbf{6 \times {}^{-}4 - ({}^{-}4)^2}$

$= \mathbf{{}^{-}24 - ({}^{+}16)}$

$= \mathbf{{}^{-}40}$

The idea that -4^2 has "implied brackets" is difficult to grasp. ie $-(4^2)$

(NB If students just type -4^2 on a calculator they will get –16)

10. In multiplications, the idea that "***an EVEN number of negatives means the answer is positive and an ODD number of negatives means a negative answer***", takes a while for some students to grasp but it makes a good class discussion time discovering this idea.

ie $^{-}4 \times {}^{-}3 \times {}^{-}5 \times 2 \times 1 = {}^{-}\mathbf{120}$ but $^{-}4 \times {}^{-}3 \times {}^{-}5 \times 2 \times {}^{-}1 = {}^{+}\mathbf{120}$

and $({}^{-}1)^{25} = {}^{-}\mathbf{1}$ but $({}^{-}1)^{26} = {}^{+}\mathbf{1}$