See my website www.mathematicalgems.weebly.com *for a PowerPoint showing the following transformation of a cuboid into 3 identical pyramids.*

VOLUME OF A PYRAMID.

Finding the volume of a PYRAMID is another good example of a topic where the teacher usually just TELLS students the formula and simply expects them to believe it without question!

Most teachers just TELL students that the volume is equal to:

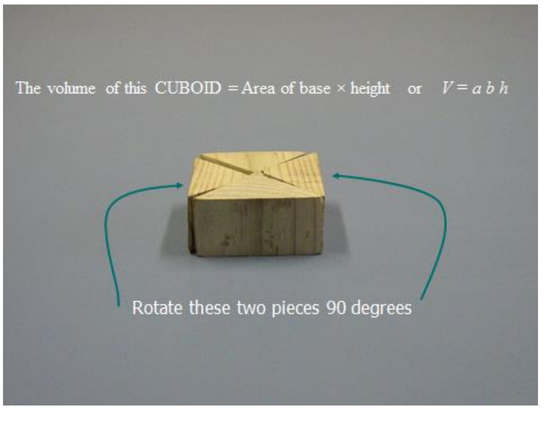
"The area of the base multiplied by the height then all divided by 3".

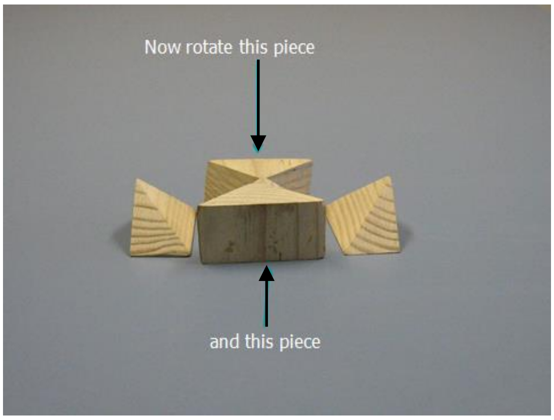
Volume = $\dfrac{\textbf{area of the base} \times \textbf{height}}{\textbf{3}}$

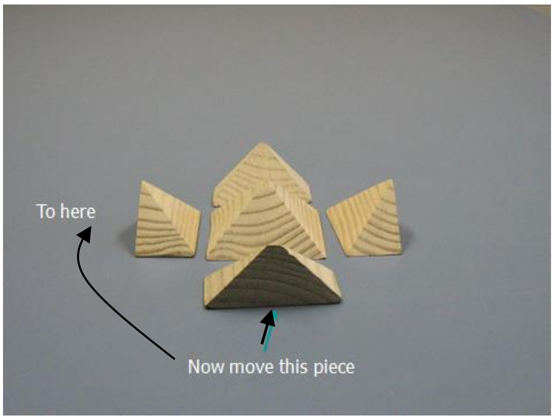
It is possible to demonstrate WHY this is true with the aid of a specially designed model consisting of 3 equally sized pyramids which may be fitted together to make a cuboid whose volume is "A**rea of the base × the height"**.

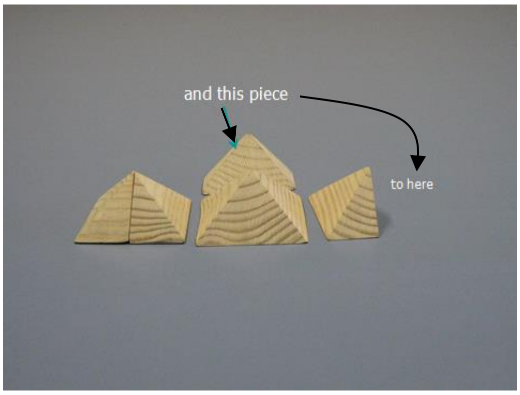
SEE MY VIDEO DEMO:

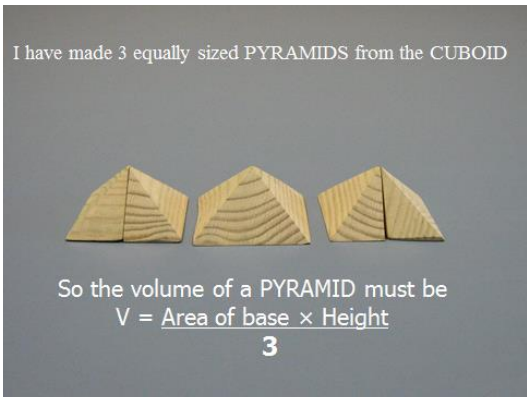
http://screencast.com/t/oligrSdlZ



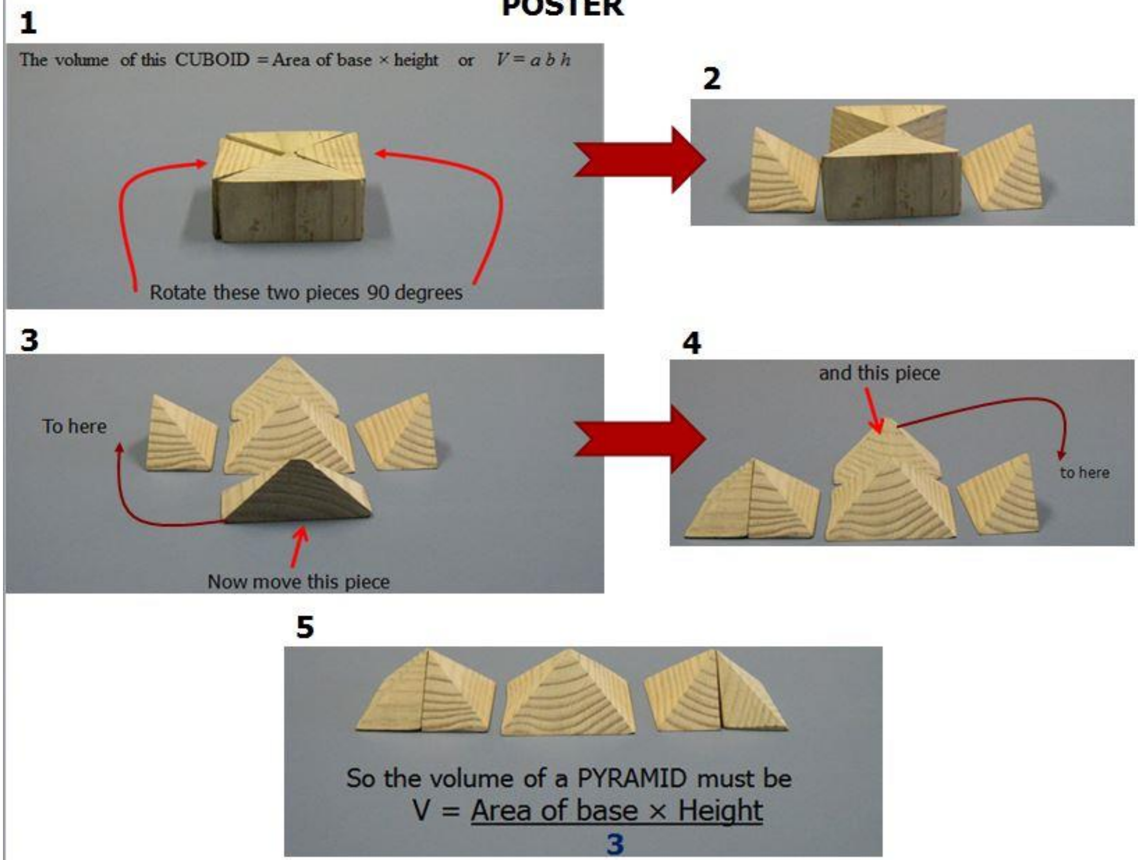

Now rotate this piece
and this piece


To here
Now move this piece


and this piece
to here


I have made 3 equally sized PYRAMIDS from the CUBOID
So the volume of a PYRAMID must be
V = Area of base × Height
3

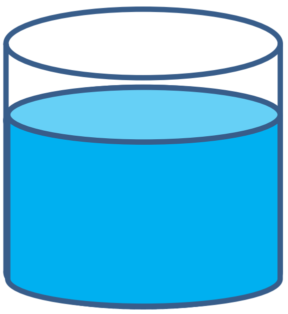
POSTER


1
The volume of this CUBOID = Area of base × height or $V = a\,b\,h$
Rotate these two pieces 90 degrees
2
3
To here
Now move this piece
4
and this piece
to here
5
So the volume of a PYRAMID must be
$V = \frac{\text{Area of base} \times \text{Height}}{3}$

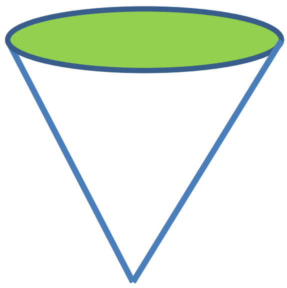
VOLUME OF A CONE.

Incidentally, we cannot split up a cylinder into 3 cones to demonstrate that the volume of a cone is $\frac{\textit{Area of base} \times \textit{height}}{3} = \frac{\pi r^2 h}{3}$

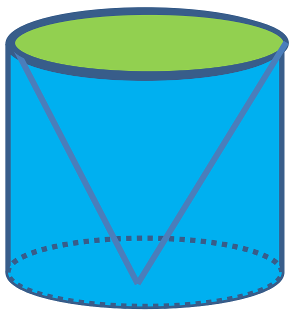
but we can use a model of a solid cone which fits into a cylinder ⅔ full of water. When we place the cone in the cylinder, the water rises to fill the rest of the cylinder, implying that the volume of the cone = ⅓ the volume of the cylinder.



Cylinder ⅔ full of water.



Solid Cone with the same base and same height as the cylinder.



When the cone is placed in the cylinder, the water level rises to fill the cylinder.

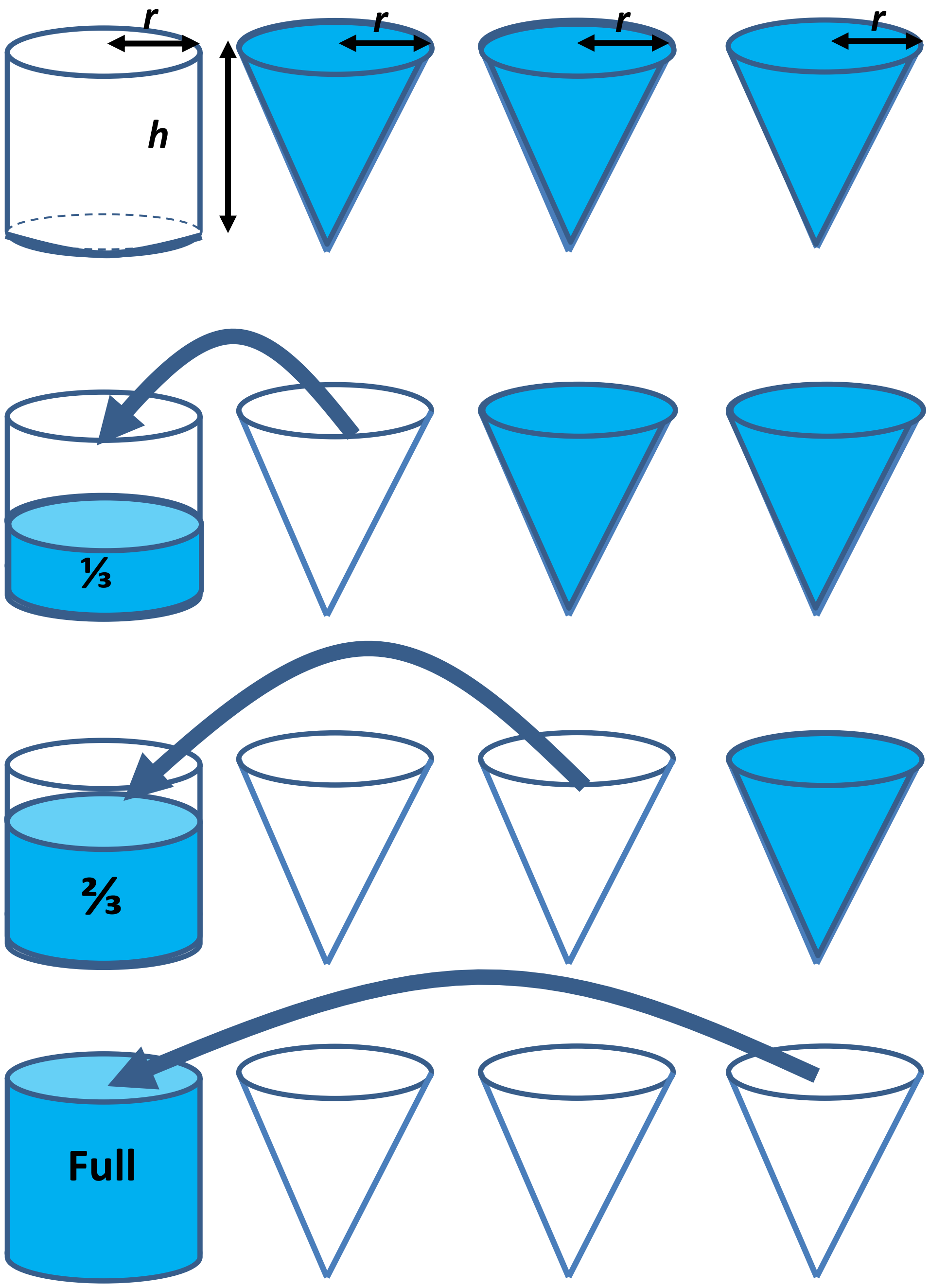
Ideally, the cylinder and cone should have the same base radius and the same height, but this could be done with a larger measuring cylinder and a small cone. We simply measure the change in volume of the water.

If the cone is open we could just fill it with water 3 times and pour it into the cylinder!

See the following demonstration/poster!

To show the **VOLUME OF A CONE** = $\frac{\pi r^2 h}{3}$

Fill the 3 cones with water and pour the contents into the cylinder one by one.

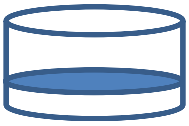


VOLUME OF A SPHERE.

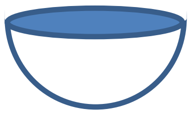
Without using “volume of revolution” methods it would be hard to show that the volume of a sphere is $V = \frac{4\pi r^3}{3}$ but we could still demonstrate it by dropping a small sphere into a cylinder of water.

Perhaps the clearest demonstration would be dropping a hemisphere of radius ***r*** into a cylinder of radius r and height r which is ⅓ full of water.

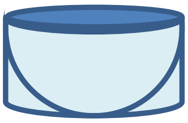
The hemisphere would have a volume of $\frac{2}{3}\pi r^3$ and so the water would fill the cylinder.



cylinder radius ***r***
and height ***r***
⅓ full of water



hemisphere
of radius ***r***



When hemisphere is placed in the cylinder the water rises from ⅓ full to completely fill the cylinder.

If it is not possible to make the radii the same, the sphere could just be put in a measuring cylinder and simply note the rise in water level.