**WHY IS 20 EQUAL TO 1?**

When I first see my Calculus class each year, I tell them I am going to show them the difference between KNOWING something and UNDERSTANDING it.

I then ask them**, “Who knows what 20 equals?”**

Most people do KNOW that 20 = 1, however, when I say: **“Who knows WHY 20 = 1?”** **NOBODY can explain to me WHY.**

I then go through some very basic work on indices and, after a few minutes, they can all explain to me ***WHY 20 equals 1****.*

They can also explain ***WHY 2 – 3 = 1***

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This is a very worthwhile exercise and I tell them that from now on, I expect them to **not just know HOW** to do problems but to **be able to explain WHY** the methods work.

This is basically how I proceed:

***We know that b3 means b×b×b***

 ***and b2 means b×b***

 ***and b1 means b***

 ***but b0 does not seem to make any sense***

 ***and b – 1  seems to make even less sense.***

***Using what we DO know: b3 × b2 means b×b×b×b×b = b5***

***So we see that we do not need to write all the b’s out, we could just generalise and put b17×b13 = b17 + 13 = b30***

***and so bn × bp = b (n + p)***

***Similarly, b5 = b×b×b×b×b = b×b = b2 by cancelling out where the b = 1***

 ***b3 b×b×b b***

***We can generalise here too so that b18 = b18 – 14 = b4***

 ***b14***

 ***and so bn = b n – p***

 ***bp***

***Now this SEEMS to be fine as long as n > p***

***Suppose we want b3***

 ***b3***

***If we use the rule above, we get an unusual answer b3 = b3 – 3 = b0 ?***

 ***b3***

***but if we work it out using the fundamental logic we get b3 = b×b×b = 1***

 ***b3 b×b×b***

***The LOGICAL conclusion is that b0 MUST be 1***

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***Similarly,*** s***uppose we want b3***

 ***b5***

***If we use the rule above, we get an unusual answer b3 = b3 – 5 = b – 2  ?***

 ***b5***

***but if we work it out using the logic we get b3 = b×b×b = 1***

 ***b5 b×b×b×b×b b2***

***The LOGICAL conclusion is that b – 2  MUST be 1***

 ***b2***

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***Now we have a meaning for “b” to the power of any INTEGER.***

***b3 means b×b×b***

***b2 means b×b***

***b1 means b***

***b0 means 1***

***b – 1 means 1***

 ***b***

***b – 2 means 1***

 ***b2***

***b – 3 means 1***

 ***b3***

***Incidentally, if we know that*** ***bn × bp = b (n + p)***

***we can consider indices which are fractions.***

***Consider 9 ½ × 9 ½  = 9 ½ + ½  = 91 but 3 × 3 = 9***

***so 9 ½  must be*** *√****9 which is 3***

***Similarly 8⅓×8⅓×8⅓ = 8 1***

 ***but 2 × 2 × 2 = 8***

 ***so 8⅓ must be the cube root of 8***

***NOTE: for most students a NUMERICAL verification is far more meaningful***

 ***than a so called “rigorous” proof.***

There are many posters like the following:

***bn×bp = bn + p***

***bn = bn – p***

***bp***

***b ½  = √b***

but I would much rather see posters like the following:

***b3 = b3 – 3 = b0 b3***

***b3 = b×b×b = 1 b3 b×b×b***

***so b0 MUST be 1***

 ***b½ ×*** ***b ½*** ***= b1***

***√b***  ***× √b = b***

***So b½ must be √b***

***b3 = b3 – 5 = b – 2  b5***

***b3 = b×b×b = 1 b5 b×b×b×b×b b 2***

***so b – 2  MUST be 1***

 ***b 2***