<u>WHY IS 2^0 EQUAL TO 1?</u>

When I first see my Calculus class each year, I tell them I am going to show them the difference between KNOWING something and UNDERSTANDING it.

I then ask them**, "Who knows what 2^0 equals?"**

Most people do KNOW that $2^0 = 1$, however, when I say:

"Who knows WHY $2^0 = 1$?" NOBODY can explain to me WHY.

I then go through some very basic work on indices and, after a few minutes, they can all explain to me ***WHY 2^0 equals 1***.

They can also explain ***WHY*** $2^{-3} = \dfrac{1}{8}$

This is a very worthwhile exercise and I tell them that from now on, I expect them to **not just know HOW** to do problems but to **be able to explain WHY** the methods work.

This is basically how I proceed:

We know that b^3 means $b\times b\times b$

and b^2 means $b\times b$

and b^1 means b

but b^0 does not seem to make any sense

and b^{-1} seems to make even less sense.

Using what we DO know: $b^3 \times b^2$ ***means*** $b\times b\times b\times b\times b = b^5$

So we see that we do not need to write all the b's out, we could just generalise and put $b^{17}\times b^{13} = b^{17+13} = b^{30}$

and so $b^n \times b^p = b^{(n+p)}$

Similarly, $\dfrac{b^5}{b^3} = \dfrac{b\times b\times b\times b\times b}{b\times b\times b} = b\times b = b^2$ ***by cancelling out where the*** $\dfrac{b}{b} = 1$

We can generalise here too so that $\dfrac{b^{18}}{b^{14}} = b^{18-14} = b^4$

$$\textit{and so} \quad \frac{b^n}{b^p} = b^{n-p}$$

Now this SEEMS to be fine as long as $n > p$

Suppose we want $\frac{b^3}{b^3}$

If we use the rule above, we get an unusual answer $\frac{b^3}{b^3} = b^{3-3} = b^0$ *?*

but if we work it out using the fundamental logic we get $\frac{b^3}{b^3} = \frac{b \times b \times b}{b \times b \times b} = 1$

The LOGICAL conclusion is that b^0 *MUST be 1*

Similarly, suppose we want $\frac{b^3}{b^5}$

If we use the rule above, we get an unusual answer $\frac{b^3}{b^5} = b^{3-5} = b^{-2}$ *?*

but if we work it out using the logic we get $\frac{b^3}{b^5} = \frac{b \times b \times b}{b \times b \times b \times b \times b} = \frac{1}{b^2}$

The LOGICAL conclusion is that b^{-2} *MUST be* $\frac{1}{b^2}$

<u>Now we have a meaning for "b" to the power of any INTEGER.</u>

b^3 *means* $b \times b \times b$

b^2 *means* $b \times b$

b^1 *means* b

b^0 *means* 1

b^{-1} *means* $\frac{1}{b}$

b^{-2} *means* $\frac{1}{b^2}$

b^{-3} *means* $\frac{1}{b^3}$

Incidentally, if we know that $b^n \times b^p = b^{(n+p)}$
we can consider ***<u>indices which are fractions.</u>***

Consider $9^{½} \times 9^{½} = 9^{½ + ½} = 9^1$ *but* $3 \times 3 = 9$
so $9^{½}$ *must be* $\sqrt{9}$ *which is 3*

Similarly $8^{⅓} \times 8^{⅓} \times 8^{⅓} = 8^1$
but $2 \times 2 \times 2 = 8$
so $8^{⅓}$ *must be the cube root of 8*

<u>NOTE</u>: for most students a NUMERICAL verification is far more meaningful than a so called "rigorous" proof.

There are many posters like the following:

$$b^n \times b^p = b^{n+p}$$

$$\frac{b^n}{b^p} = b^{n-p}$$

$$b^{\frac{1}{2}} = \sqrt{b}$$

but I would much rather see posters like the following:

$$\frac{b^3}{b^3} = b^{3-3} = b^0$$

$$\frac{b^3}{b^3} = \frac{b \times b \times b}{b \times b \times b} = 1$$

so b^0 *MUST be 1*

$$\frac{b^3}{b^5} = b^{3-5} = b^{-2}$$

$$\frac{b^3}{b^5} = \frac{b \times b \times b}{b \times b \times b \times b \times b} = \frac{1}{b^2}$$

so b^{-2} *MUST be* $\frac{1}{b^2}$

$$b^{\frac{1}{2}} \times b^{\frac{1}{2}} = b^1$$

$$\sqrt{b} \times \sqrt{b} = b$$

So $b^{\frac{1}{2}}$ *must be* $\sqrt{b}$