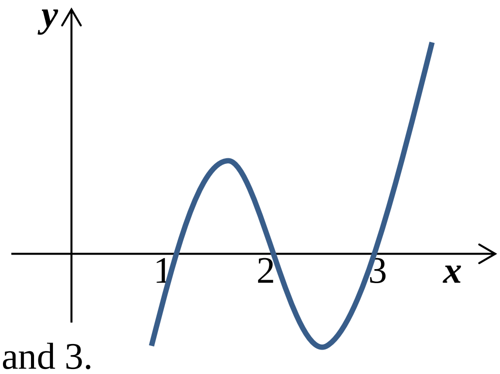
THE FACTOR AND REMAINDER THEOREMS.

It is sad that for a huge majority of people these theorems consist merely of two almost mysterious rules!
You will find that students can quote things like ***"If $(x - 2)$ is a factor of $f(x)$ then $f(2) = 0$"*** and ***"If $f(x)$ is divided by $(x - 3)$ then the remainder is $f(3)$"*** but ask them WHY and they will be at a loss for words.

The Factor Theorem:



Suppose $f(x) = (x - 1)(x - 2)(x - 3)$
This represents a cubic graph which crosses the x axis at $x = 1, 2$ and 3

Expanding $f(x) = x^3 - 6x^2 + 11x - 6$

Obviously the only 3 times $f(x)$ can be zero are where the graph crosses the x axis at $x = 1$, 2 and 3.

Comparing the two versions of $f(x)$
$f(x) = x^3 - 6x^2 + 11x - 6$
$f(x) = (x - 1)(x - 2)(x - 3)$

If we substitute $x = 1$ then $f(1) = (0)\times(1 - 2)\times(1 - 3) = 0$
If we substitute $x = 2$ then $f(2) = (2 - 1)\times(0)\times(2 - 3) = 0$
If we substitute $x = 3$ then $f(3) = (3 - 1)\times(3 - 2)\times(0) = 0$

For any polynomial $g(x)$ it should be obvious to say that if $g(5) = 0$ then $g(x)$ MUST equal $(x - 5)\times$(something)

This would make a suitable poster for a classroom:

FACTOR THEOREM EXPLAINED.
Let $F(x) = x^3 + ax^2 + bx + c$
If we try $x = 4$ and we find that $F(4) = 0$ then $F(x)$ must equal $(x - 4)\times($ *something* $)$
ie If $F(4) = 0$ then $(x - 4)$ is a factor of $F(x)$

The RemainderTheorem:

Consider this ***long division:***

$$\begin{array}{r|l} & x^2 + 3x + 7 \\ \hline x-2 & x^3 + x^2 + x + 4 \\ & \underline{x^3 - 2x^2} \\ & \quad 3x^2 + x \\ & \quad \underline{3x^2 - 6x} \\ & \qquad 7x + 4 \\ & \qquad \underline{7x - 14} \\ & \qquad 0 + 18 \end{array}$$

This could be written as:

$$\frac{x^3 + x^2 + x + 4}{(x-2)} = x^2 + 3x + 7 + \frac{18}{(x-2)}$$

OR

$$x^3 + x^2 + x + 4 = (x-2)(x^2 + 3x + 7) + 18$$

We say that *18* is the REMAINDER

Consider these two versions of $f(x)$:

(a) $f(x) = x^3 + x^2 + x + 4$

(b) $f(x) = (x-2)(x^2 + 3x + 7) + 18$

If we substitute $x = 2$ ***in (a) we get*** $f(2) = 8 + 4 + 2 + 4 = 18$

If we substitute $x = 2$ ***in (b) we get*** $f(2) = (0)\times(4 + 6 + 7) + 18 = 18$

Clearly the **remainder** after dividing by $(x-2)$ is simply $f(2) = 18$

If we were to divide the same function by $(x-1)$ ***we could write it as:***

$f(x) = x^3 + x^2 + x + 4 = (x-1)\times(\text{ something }) + R$

Substituting $x = 1$: $1 + 1 + 1 + 4 = (0)\times(\text{ something }) + R$

The remainder R would be $f(1) = 1 + 1 + 1 + 4 = 7$

Generally, if $f(x)$ is divided by $(x-a)$

Then $f(x) = (x-a)\times(\text{ something}) + R$ where R is the remainder.

so on substituting $x = a$ we would get $f(a) = (0)\times(\text{something}) + R$

This would make a suitable poster for a classroom:

REMAINDER THEOREM EXPLAINED.

Let $F(x) = x^3 + ax^2 + bx + c$

If we divide $F(x)$ ***by*** $(x-5)$ ***and the remainder is 12***

then $F(x)$ must equal $(x-5)\times(\text{ something }) + 12$

so $F(5) = (0)\times(\text{ something }) + 12 = 12 =$ ***the remainder.***