## LOGARITHMS.

This topic has been a total mystery to students for many years because, for most people, it merely reduces to the application of certain rules with little understanding of what they mean.

## A LOGARITHM is just an INDEX.

Consider the index equation $\mathbf{2}^{3}=\mathbf{8}$
The following are equivalent statements:

$$
\begin{aligned}
& \text { The index }=3 \\
& \text { The } \log =3 \\
& \log _{2} 8=3 \\
& \text { index } 8=3
\end{aligned}
$$

A light comes on in some students minds when they realise this simple fact.
The so called "Three Log Rules" have always caused problems because they seem to be so obscure.
Algebraic proofs of the rules never seem to get through to students.

## 1. $\log (x y)=\log x+\log y$

This means that when you multiply two numbers you add the indices (ie logs)
Suppose $x=c^{a}$ and $y=c^{b}$
$\log (x \times y)$ means: what is the index of $x \times y$
ie what is the index of $c^{a} \times c^{b}$

$$
\text { but } c^{a} \times c^{b}=c^{(a+b)}
$$

so the index of $x \times y$ is $a+b$
ie the index of $x \times y$ equals the (index of $x$ ) plus (the index of $y$ )
or more explicitly, the $\log$ of $x \times y=$ the $\log$ of $x+$ the $\log$ of $y$
Expressing this in words seems to be the best thing to do:
When you multiply two numbers you add the indices (ie logs)
2. $\log \left(\frac{x}{y}\right)=\log x-\log y$

This means that when you divide two numbers you subtract the indices (logs)
Suppose $x=c^{a}$ and $y=c^{b}$

$$
\frac{x}{y}=\frac{c^{a}}{c^{b}}=c^{(a-b)}
$$

$\log$ of $\frac{x}{y}$ means: what is the index of $\frac{x}{y}$
ie what is the index of $\frac{c^{a}}{c^{b}}$
ie what is the index of $c^{(a-b)}$
and clearly the index is $a-b$
ie the index of $\underline{x}$ equals the (index of $x$ ) minus (the index of $y$ )
$y$
or more explicitly the $\log$ of $\underline{x}=$ the $\log$ of $x-$ the $\log$ of $y$
$y$
Again, expressing this in words seems to have the most success:
When you divide two numbers you subtract the indices (logs)

## 3. $\log x^{n}=n \log x$

This is just an extension of law 1 and becomes more meaningful when we consider:

$$
\begin{aligned}
& \log x^{3}=\log x \cdot x \cdot x \\
& \quad=\log x+\log x+\log x \\
& \quad=3 \log x
\end{aligned}
$$

## CHANGING THE BASE OF LOGARITHMS.

| LOGICAL METHOD | FORMULA METHOD |
| :---: | :---: |
| Find the value of $\log _{2} 7$ | Find the value of $\log _{2} 7$ |
| Let $x=\log _{2} 7$ | Using the formula $\log _{b} x=\underline{\log _{a}} \underline{\log _{a}} \frac{x}{b}$ |
| So $2^{x}=7$ |  |
| Find $\boldsymbol{\operatorname { l o g }}_{10}$ of both sides : | $\log _{2} 7=\frac{\log _{10}}{\log _{10}} \frac{7}{2}$ |
| $\log _{10} 2^{x}=\log _{10} 7$ | $=2.807$ |
| $x \log _{10} 2=\log _{10} 7$ |  |
| $x=\frac{\log _{10}}{\log _{10}} \frac{7}{2}$ |  |
| $=2.807$ |  |
|  | Obviously, this formula method is a lot quicker but speed is nothing to do with "understanding". <br> For instance, you could tell a student that the formula is: $\log _{b} x=\frac{\log _{a}}{\log _{a}} \frac{b}{x}$ <br> and the student would happily substitute the values obtaining an answer which they would think is quite acceptable because the formula said so! |

