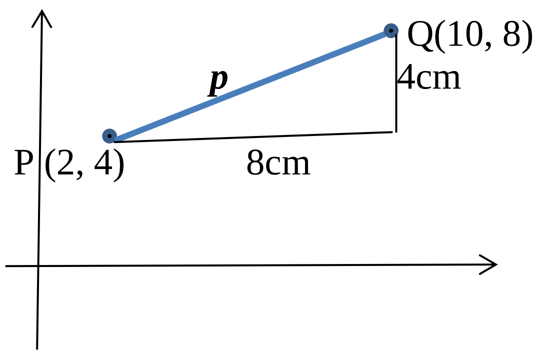
COORDINATE GEOMETRY.

This is a completely simple, logical topic and should not be taught using all those unnecessary formulae involving x_1, y_1, x_2, y_2.

In fact the only formula we need to use is $y = mx + c$

Examples.

Suppose P is (2, 4) and Q is (10, 8)



(i) Find the length of PQ

DRAW A SMALL SKETCH and use Pythagoras' Theorem!

$p^2 = 8^2 + 4^2$

$p^2 = 64 + 16$

$p^2 = 80$

$p \approx 8.94cm$

(ii) Find the gradient of PQ

Just look at the sketch above! $grad = \frac{4}{8} = \frac{1}{2}$

(iii) Find the EQUATION of PQ

The basic equation is $y = mx + c$

We just found the gradient $m = ½$

So $y = \frac{x}{2} + c$

The graph goes through (2, 4) so subs $x = 2, y = 4$

$4 = 1 + c$

$3 = c$

The equation of PQ is $y = ½ x + 3$

(iv) Find M, the mid-point of PQ

M is the average of (2, 4) and (10, 8)

ie average 2 and 10 = $\frac{12}{2}$ = 6 and average of 4 and 8 = 6

Mid point is M = (6, 6)

The above techniques are just very basic logic.
The following formulae should be made completely redundant:

Dist = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Gradient = $\dfrac{y_2 - y_1}{x_2 - x_1}$

Equation of line $\quad y - y_1 = m(x - x_1)$

OR $\quad \dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$

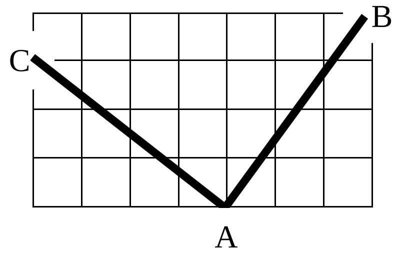
Mid Point = $\left(\dfrac{x_1 + x_2}{2} \,,\, \dfrac{y_1 + y_2}{2} \right)$

Whether students/tutors/teachers realise it or not, the use of these formulae does not promote proper "thinking" and "understanding".

N.B. If you told students that the equation of a line was $y + y_1 = m(x + x_1)$ then students would cheerfully use it just as "confidently" and happily as they would use the correct version.
They may THINK that they understand it but clearly they only KNOW how to substitute numbers into an equation whether the equation is correct or not! <u>The "formula" replaces understanding!</u>

<u>Parallel lines</u> : obviously have the same gradient.
eg $\;y = \dfrac{3}{4}x + 5\;$ is parallel to $y = \dfrac{3}{4}x - 9$

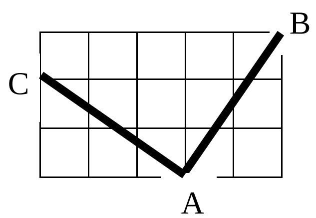
<u>Perpendicular Lines</u> : we need to show examples as follows



Grad AB = $\dfrac{4}{3}$

Grad AC = $\dfrac{-3}{4}$

Students will easily realise that the product of perpendicular gradients is -1

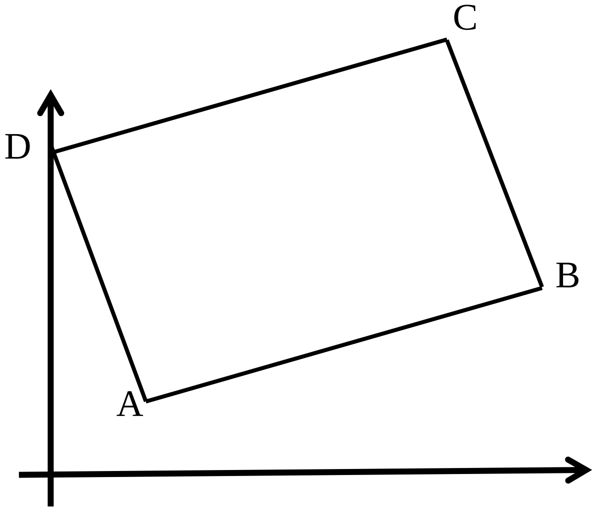


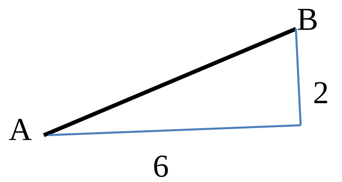
Grad AB = $\dfrac{3}{2}$

Grad AC = $\dfrac{-2}{3}$

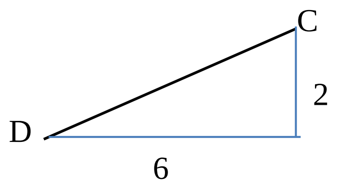
The following is a model answer for a suitable teaching example for this topic using the logical methods as described above.

Using coordinate geometry methods find the properties of the quadrilateral given by A(2, 2), B(8, 4), C(6, 10) and D(0, 8) . As a result of the properties you have found, state what type of quadrilateral this is.
Also find the intersection point of the diagonals.

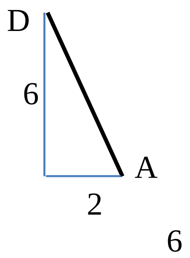




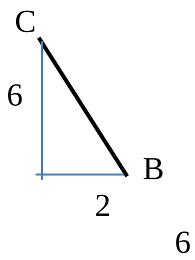
$AB^2 = 6^2 + 2^2$
$AB = \sqrt{40} \approx 6.32$



$DC^2 = 6^2 + 2^2$
$DC = \sqrt{40} \approx 6.32$



$DA^2 = 6^2 + 2^2$
$DA = \sqrt{40} \approx 6.32$



$BC^2 = 6^2 + 2^2$
$BC = \sqrt{40} \approx 6.32$

Therefore all 4 sides are equal in length.

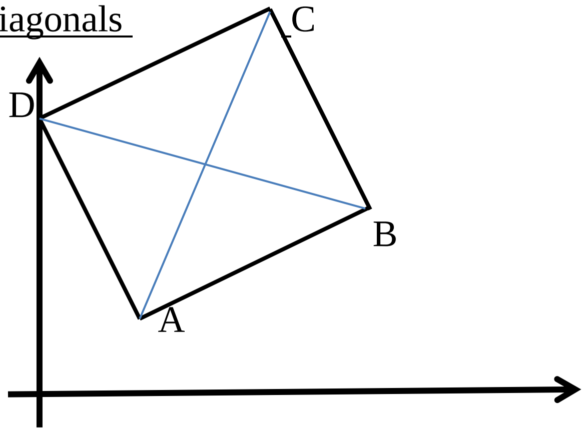
Using the above sketches:

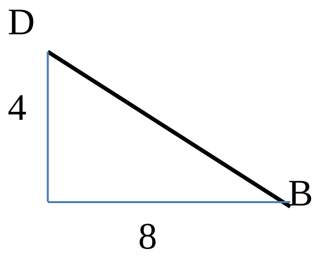
$$\left.\begin{array}{l}\text{Gradient of } AB = \frac{2}{6} = \frac{1}{3} \\ \text{Gradient of } DC = \frac{2}{6} = \frac{1}{3}\end{array}\right\}$$ **So AB is parallel to DC**

$$\left.\begin{array}{l}\text{Gradient of } AD = \frac{6}{2} = 3 \\ \text{Gradient of } BC = \frac{6}{2} = 3\end{array}\right\}$$ **So AD is parallel to BC**

Considering the gradients of **<u>adjacent sides</u>** ***Grad AB* × *Grad BC* = – *1***
this means that the **lines are perpendicular**.
This means all the angles are 90^0, all the sides are equal and opposite sides are parallel so **ABCD MUST be a SQUARE.**

<u>Diagonals</u>





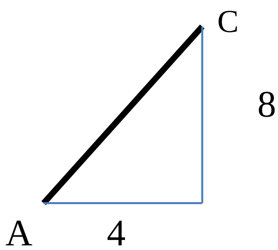
Grad of DB = $\frac{-1}{2}$

Equation of DB is of the form $y = mx + c$ where $m = -\frac{1}{2}$

and it goes through (0, 8) so substituting:

$$8 = -\frac{1}{2} \times 0 + c \quad \text{so} \quad c = 8$$

DB is $y = -\frac{1}{2}x + 8$



Grad of AC = ***2***

Equ of AC is of the form $y = mx + c$
where $m = 2$ and it goes through (2, 2)
so substituting:

$$2 = 2\times 2 + c \quad \text{so} \quad c = -2$$

Equ AC is $y = 2x - 2$

The diagonals intersect when $2x - 2 = -\frac{1}{2}x + 8$

Mult b.s. by 2: $4x - 4 = -x + 16$

$$5x = 20$$

$$x = 4 \quad \text{and} \quad y = 6$$