

How do I prove by induction ?

Most students find induction to be very confusing!

I think a very simple example would be a good idea.

I realise we can easily show, using simple arithmetical series that the sum of the first **n** counting numbers can be written as:

$$\text{Sum } S_n = \frac{n(n+1)}{2}$$

Let's prove this by induction!

The first thing to do is to check that it is ever true!

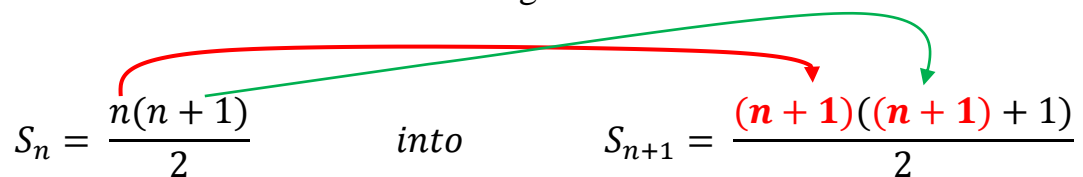
If $n = 1$ the formula comes to:

$$S_1 = \frac{1(1+1)}{2} = 1$$

The idea of induction is now to show that if it is true for **n** numbers then the **same formula** holds for **n+1** numbers.

This is the part students find it hard to grasp.

By adding the next number, which is **(n + 1)**, to the formula $\frac{n(n+1)}{2}$ we have to show that the formula changes from:



The diagram illustrates the substitution process in the induction step. It shows the formula for S_n on the left, followed by the word "into", and then the formula for S_{n+1} on the right. A red arrow points from the n in the numerator of the first formula to the $(n+1)$ in the numerator of the second formula. A green arrow points from the $(n+1)$ in the denominator of the first formula to the $((n+1)+1)$ in the denominator of the second formula. The terms $(n+1)$ and $((n+1)+1)$ in the second formula are highlighted in red.

$$S_n = \frac{n(n+1)}{2} \quad \text{into} \quad S_{n+1} = \frac{(n+1)((n+1)+1)}{2}$$

.....you see we have just replaced **n** with **n + 1** in the above formula!

This is how we show it...

If the sum of n terms is $\frac{n(n+1)}{2}$

...then the sum of $\mathbf{n + 1}$ terms should be... $\frac{n(n+1)}{2} + (\mathbf{n + 1})$

$$= (n + 1) \left(\frac{n}{2} + 1 \right)$$

$$= (n + 1) \left(\frac{n}{2} + \frac{2}{2} \right)$$

$$= (n + 1) \left(\frac{(n + 2)}{2} \right)$$

$$= \frac{(\mathbf{n + 1})(\mathbf{n + 1} + 1)}{2}$$

So we have shown the formula is true when $\mathbf{n = 1}$ and we have shown that if it is true for n numbers then it is true for $\mathbf{n+1}$ numbers.

This proves that it is always true.