How do I prove by induction ?

Most students find induction to be very confusing!

I think a very simple example would be a good idea.

I realise we can easily show, using simple arithmetical series that the sum of the first **n** counting numbers can be written as:

$$Sum\ S_n = \frac{n(n+1)}{2}$$

Let's prove this by induction!

The first thing to do is to check that it is ever true!
If n = 1 the formula comes to:

$$S_1 = \frac{1(1+1)}{2} = 1$$

The idea of induction is now to show that if it is true for **n** numbers then the **same formula** holds for **n+1** numbers.

This is the part students find it hard to grasp.

By adding the next number, which is **(n + 1)**, to the formula $\frac{n(n+1)}{2}$ we have to show that the formula changes from:

$$S_n = \frac{n(n+1)}{2} \qquad into \qquad S_{n+1} = \frac{(\boldsymbol{n+1})((\boldsymbol{n+1})+1)}{2}$$

…….you see we have just replaced $\boldsymbol{n}$ with $\boldsymbol{n+1}$ in the above formula!

This is how we show it…

If the sum of n terms is $\frac{n(n+1)}{2}$

…then the sum of ***n* + 1** terms should be… $\frac{n(n+1)}{2} + (n+1)$

$$= (n+1)\left(\frac{n}{2}+1\right)$$

$$= (n+1)\left(\frac{n}{2}+\frac{2}{2}\right)$$

$$= (n+1)\left(\frac{(n+2)}{2}\right)$$

$$= \frac{(\boldsymbol{n}+\mathbf{1})((\boldsymbol{n}+\mathbf{1})+1)}{2}$$

So we have shown the formula is true when **n = 1** and we have shown that if it is true for n numbers then it is true for **n+1** numbers.
This proves that it is always true.