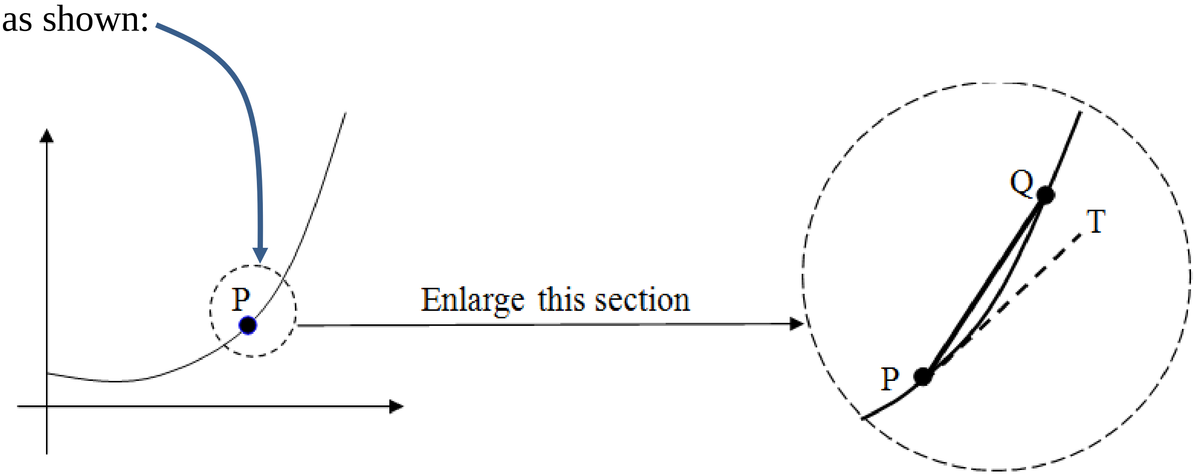
What is the difference between Δy over Δx and dy over dx

When I first started to learn Calculus we used Δ**x** and Δ**y** a lot but in more recent times I have rarely seen Δ**x** and Δ**y** probably because of the obvious confusion it causes for young students.

To find the gradient of a curve, we imagine a microscopic section of the curve as shown:

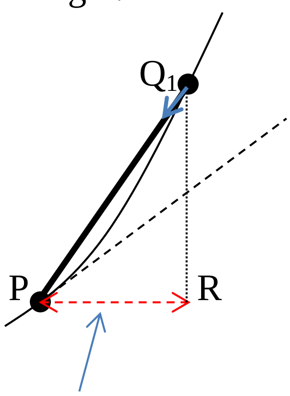


<u>Fundamental Idea</u>: The steepness of tangent PT is approximately equal to the steepness of a chord PQ (where Q is a point on the curve very close to P).

This approximation gets closer to the gradient of the tangent at P as Q moves closer and closer to P.

In all my teaching I have use **"h"** instead of **"Δx"** as shown in these diagrams…

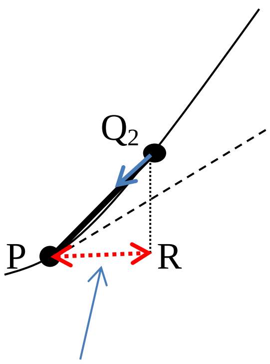
Fig 1.



Δx = h = 0.01

Grad $PQ_1 = \dfrac{Q_1R}{PR}$

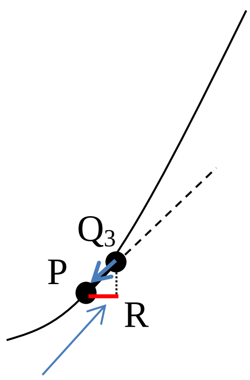
Fig 2.



Δx = h = 0.001

Grad $PQ_2 = \dfrac{Q_2R}{PR}$

Fig 3.

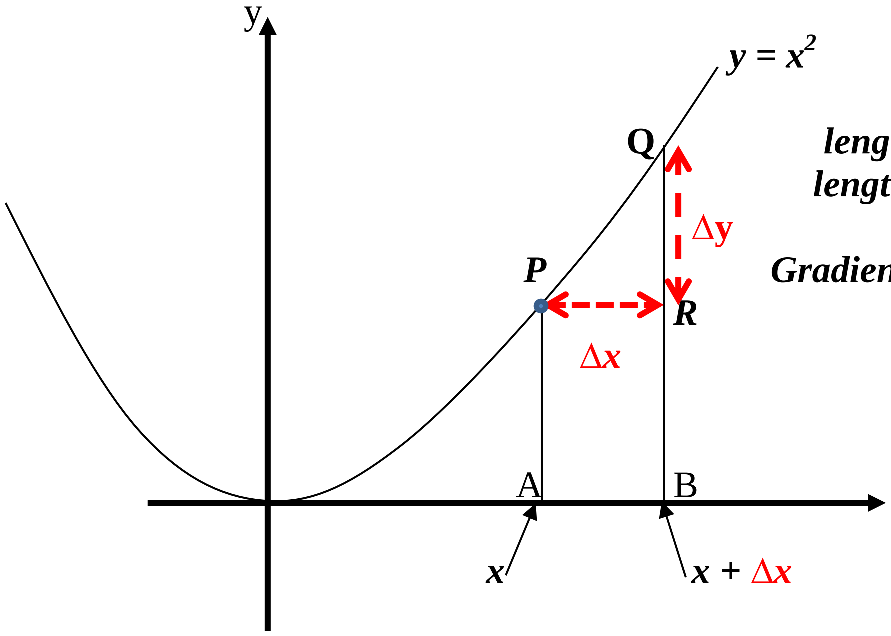


Δ**x** = h = 0.00001

Grad $PQ_3 = \dfrac{Q_3R}{PR}$

This **“geometrical idea”** has to be changed into **“mathematical symbols”** so that we can find the numerical values of these gradients and work out what value they are approaching.

The “old fashioned” way using Δ**x** and Δ**y** looked like this…



length of PA $= x^2$

length of **QB** $= (x + \Delta x)^2$

Gradient of PQ $= \dfrac{QR}{PR} = \dfrac{\Delta y}{\Delta x}$

$$= \frac{QB - PA}{PR}$$

$$= \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{\Delta x(2x + \Delta x)}{\Delta x}$$

$$= 2x + \Delta x$$

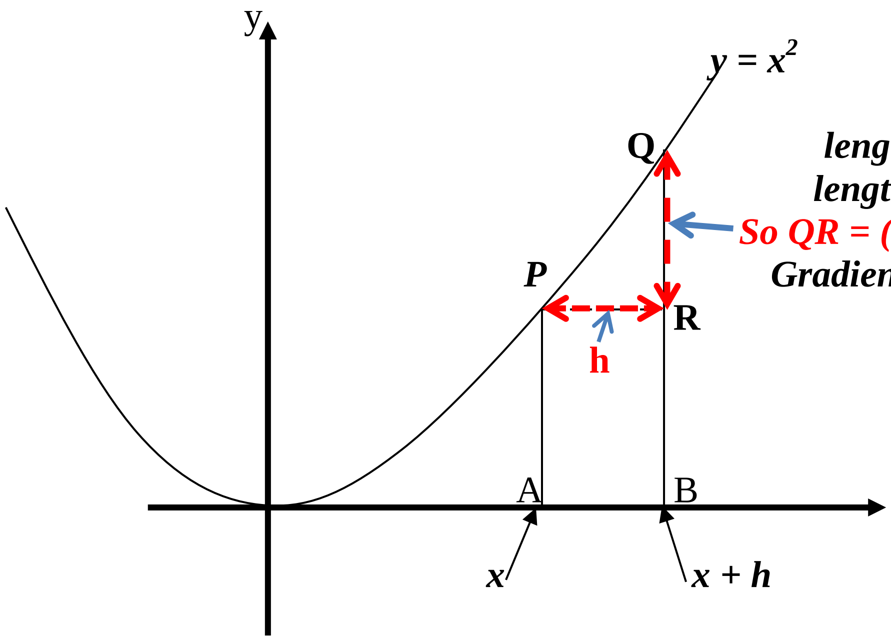
Then we say that as $\Delta x \to 0$ the gradient AT POINT P is just **2x**

and the SYMBOL used for the gradient was $\dfrac{dy}{dx} = 2x$

This symbol was not considered to be an actual fraction!
but $\dfrac{\Delta y}{\Delta x}$ is a fraction and equal to $\dfrac{QR}{PR}$ in the diagram above.

Now I have not written this out as above for many years and I find it very awkward and cumbersome!

Now for the “modern” way using “**h**” instead of “Δ*x*” (and we don’t even use a symbol for “Δ*y*”) …



length of PA = x^2

length of QB = $(x + h)^2$

So QR = $(x + h)^2 - x^2$

Gradient of **PQ** = $\dfrac{\mathbf{QR}}{\mathbf{PR}}$

$$= \frac{QB - PA}{PR}$$

$$= \frac{(x + h)^2 - x^2}{h}$$

$$= \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \frac{h(2x + h)}{h}$$

$$= 2x + h$$

Then we say that as: $\mathbf{h \to 0}$ the gradient AT POINT P is just **2x**

and the SYMBOL used for the gradient is usually " y' = 2x " although we still use $\dfrac{dy}{dx}$ **= 2x** of course.

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