**THE FACTOR AND REMAINDER THEOREMS.**

It is sad that for a huge majority of people these theorems consist merely of two

almost mysterious rules!

You will find that students can quote things like ***“If (x – 2) is a factor of f(x) then f(2) = 0”*** and ***“If f(x) is divided by (x – 3) then the remainder is f(3)”***

but ask them WHY and they will be at a loss for words.

**The Factor Theorem:**

***y***

Suppose ***f(x) = (x – 1)(x – 2)(x – 3)***

This represents a cubic graph which crosses

the ***x*** axis at ***x = 1, 2*** and ***3***

Expanding ***f(x) = x3 – 6x2 + 11x – 6***

1 2 3 ***x***

Obviously the only 3 times ***f(x***) can be zero

are where the graph crosses the ***x*** axis at ***x*** = 1, 2 and 3.

Comparing the two versions of ***f(x)***

***f(x) = x3 – 6x2 + 11x – 6***

***f(x) = (x – 1)(x – 2)(x – 3)***

***If we substitute x = 1 then f(1) = (0)×(1 – 2)×(1 – 3) = 0***

***If we substitute x = 2 then f(2) = (2 – 1 )×(0)×(2 – 3) = 0***

***If we substitute x = 3 then f(3) = (3 – 1)×(3 – 2)× (0) = 0***

***For any polynomial g(x) it should be obvious to say that if g(5) = 0***

***then g(x) MUST equal (x – 5)×(something)***

This would make a suitable poster for a classroom:

**FACTOR THEOREM EXPLAINED.**

Let ***F(x) = x3 + ax2 + bx + c***

If we try ***x = 4*** and we find that ***F(4) = 0***

then ***F(x)*** must equal ***(x – 4)×( something )***

***ie If F(4) = 0 then (x – 4) is a factor of F(x)***

**The RemainderTheorem:**

Consider this ***long division:***

This could be written as:

***x3 + x2 + x + 4***  = ***x2 + 3x + 7 + 18***

***(x – 2)***  ***(x – 2)***

OR

***x3 + x2 + x + 4 = (x – 2)( x2 + 3x + 7) + 18***

**We say that *18* is the REMAINDER**

***x2 + 3x + 7***

***x – 2 x3 + x2 + x + 4***

***x3 – 2x2***

***3x2 + x***

***3x2 – 6x***

***7x + 4***

***7x – 14***

***0 + 18***

Consider these two versions of ***f(x)*** :

***(a) f(x)*** = ***x3 + x2 + x + 4***

***(b) f(x) = (x – 2)( x2 + 3x + 7) + 18***

***If we substitute x = 2 in (a) we get f(2) = 8 + 4 + 2 + 4 = 18***

***If we substitute x = 2 in (b) we get f(2) = (0)×(4 + 6 + 7) + 18 = 18***

Clearly the **remainder** after dividing by ***(x – 2)*** is simply ***f(2)*** = 18

If we were to divide the same function by ***(x – 1) we could write it as:***

***f(x)*** = ***x3 + x2 + x + 4 = (x – 1)×( something ) + R***

Substituting ***x = 1: 1 + 1 + 1 + 4 = (0)×( something ) + R***

The remainder ***R*** would be ***f(1) = 1 + 1 + 1 + 4 = 7***

Generally, if ***f(x)*** is divided by ***(x – a)***

Then ***f(x) = (x – a)×( something) + R*** where ***R*** is the remainder.

so on substituting ***x = a*** we would get ***f(a) = (0)×(something) + R***

This would make a suitable poster for a classroom:

**REMAINDER THEOREM EXPLAINED.**

Let ***F(x) = x3 + ax2 + bx + c***

If we divide ***F(x) by (x – 5) and the remainder is 12***

then ***F(x)*** must equal ***(x – 5)×( something ) + 12***

***so F(5) = (0)×( something ) + 12 = 12 = the remainder.***