**FINDING THE EQUATION OF A TANGENT TO A CURVE FROM A POINT NOT ON THE CURVE.**

**1. Find the equation of the tangent to *y = x2* from the point (2, 3)**

****

P

(2, 3)

In questions like this we do not know the coordinates of the point P where the tangent touches the curve.

If we let the ***x*** coordinate of P be ***a*** then the ***y*** coordinate will be ***a2 (ie y = x2)***

Using the gradient triangle drawn above we can say that the gradient of the tangent using points (2, 3) and (***a, a2***) will be ***3 – a2***

 ***2 – a***

We can find another value for the gradient at P by differentiating ***y = x2***

Gradient ***y' = 2x*** and if ***x = a*** at P then the gradient of the tangent is ***2a***

Equating these two values we get : ***2a = 3 – a2***

 ***2 – a***

 ***So that 2a(2 – a) = 3 – a2***

 ***4a – 2a2 = 3 – a2***

 ***0 = a2 – 4a + 3***

 ***0 = (a – 1)(a – 3)***

 ***a = 1 OR a = 3***

***This means there are TWO possible tangents from (2, 3) to the curve.***

***If a = 1 then P would be (1, 1), the gradient would be m = 2 and using y = mx + c substituting m = 2, x = 1, y = 1 we get c = –1***

***The tangent drawn in the above diagram is y = 2x – 1***

***If a = 3 then P would be (3, 9), the gradient would be m = 6 and using y = mx + c substituting m = 6, x = 3, y = 9 we get c = – 9***

***The other tangent drawn in the diagram below is y = 6x – 9***

***(3, 9)***



***(1, 1)***

***y = 2x – 1***

***y = 6x – 9***

(2, 3)

Q

P

This shows the TWO tangents from (2, 3) to the parabola ***y = x2***

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**2. Find the equation of the tangents to *y = 3x – x2* from the point (1, 3)**



P ***(a, 3a – a2)***

(1, 3)

Let P be the point where the tangent from (1, 3) touches the curve so the coordinates of P, are ***(a, 3a – a2).***

Using the gradient triangle shown, the gradient of the tangent in terms of ***a*** is:

***–*** ***(3 – 3a + a2)***

 ***(a – 1*** ***)***

***Differentiating y' = 3 – 2x = 3 – 2a so equating these forms:***

***3 – 2a =***  ***–*** ***(3 – 3a + a2)***

 ***(a – 1*** ***)***

***3a – 3 – 2a2 + 2a = 3a – 3 – a2***

 ***0 = a2 – 2a***

***So a = 0 or 2***

***If a = 2 then P is (2, 2) and the tangent equation is y = – x + 4***

***If a = 0 then P is (0, 0) and the other tangent equation is y = 3x (see dotted line )***

**3. Find the equation of the tangents to *y = x3* from the point (2, 4)**

 It will become apparent that there are actually THREE tangents!

 Consider the tangent drawn below:



***P(a, a3)***

***(2, 4)***

Let the coordinates of the point where the tangent meets the curve be ***(a, a3)***

Using the gradient triangle, the gradient of the tangent is ***4 – a3***

 ***2 – a***

Differentiating, we get ***y' = 3x2 = 3a2***

Equating these two expressions:

 ***3a2 =*** ***4 – a3***

 ***2 – a***

***6a2 – 3a3 = 4 – a3***

 ***0 = 2a3 – 6a2 + 4***

 ***0 = a3 – 3a2 + 2***

 ***0 = (a – 1)(a2 – 2a – 2)***

***So that a = 1 or 1±√3***

***If a = 1, P is (1, 1) the gradient is 3 and the tangent is y = 3x – 2***

***If a = 1+√3, P is (1+√3, 20.39) the grad is 22.4 and the tan is y ≈ 22.4x – 40.8***

***If a = 1–√3, P is (1–√3, –0.39) the grad is 1.6 and the tan is y ≈ 1.6x –0.78***



**4. Find the equation of the tangent to *y = x ½*– 2 from the point (–4, –2)**

The graph is like this diagram…



P (***a, a ½ – 2*** )

**(–4, –2)**

Let the tangent meet the curve at ***x = a so y = a ½ – 2***

From the gradient triangle, the gradient of the tangent = ***a*½ – 2 + 2** = ***a*½**

 ***a + 4*** ***a + 4***

If ***y = x ½*– 2** then ***y' = 1 so at x = a, the gradient of the tan is = 1***

 ***2 x½  2 a½***

Equating these two expressions we get: ***a*½ =  *1***

 ***a + 4*** ***2a½***

 hence ***2a = a + 4***

 so that ***a = 4***

The coordinates of P are ***(4, 0)***

The gradient of the tangent ***= 1***

 ***4***

The equation of the tangent is of the form ***y = mx + c***

So substituting ***x = 4, y = 0, m = ¼*** we get ***0 = 1 + c so c = –1***

The equation of the tangent is ***y*** = ***¼x – 1***

***(4, 0)***



**(–4, –2)**

**4. Find the equation of the tangents to *y = x4* – *6x2*  from the point (0, 3)**

Let the tangent meet the curve at ***x = a so y = a4* – *6a2***

From the gradient triangle, the gradient of the tangent = ***a4* – *6a2*  – 3**

 ***a***

If ***y = y = x4* – *6x2*** then ***y' = 4x3 – 12x***

so at ***x = a***, the gradient of the tangent is = ***4a3 – 12a***

Equating these two expressions we get: ***a4* – *6a2*  – 3 *= 4a3 – 12a***

 ***a***

so ***a4 – 6a2 – 3 = 4a4 – 12a2***

hence ***0 = 3a4 – 6a2 + 3***

 ***0 = 3(a2 – 1)(a2 – 1)***

 So ***a = 1 or*** ***– 1***

If ***a = 1:***

The coordinates of P are ***(1, –5)***

The gradient of the tangent ***= –8***

The equation of the tangent is of the form ***y = mx + c***

So substituting ***x = 1, y = –5, m = –8***  we get ***–5 = –8 + c so c = 3***

The equation of the tangent is ***y*** = ***–8x + 3***

If ***a = –1:***

The coordinates of P are ***(–1, –5)***

The gradient of the tangent ***= +8***

The equation of the tangent is of the form ***y = mx + c***

So substituting ***x = –1, y = –5, m = +8***  we get ***–5 = –8 + c so c = 3***

The equation of the tangent is ***y*** = ***+8x + 3***



(***–1***, ***–5***)

 (1, ***–5***)

 (0, 3)